

Volume I.

Chapter III

ASTRO-
PHOTOMETRY

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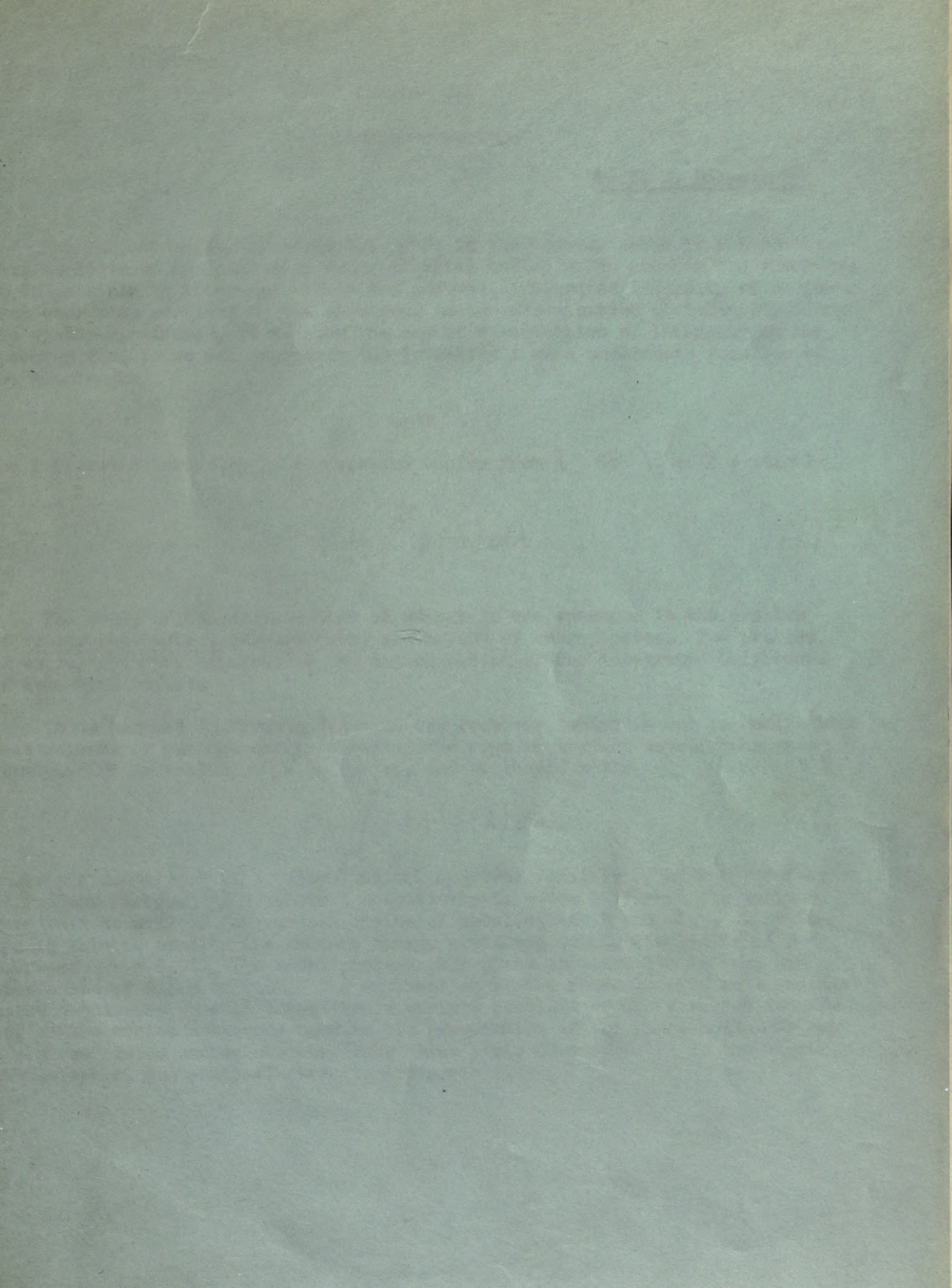
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Chapter III
Astrophotometry

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The subject matter of astrophotometry in the broader sense is a quantitative analysis of the radiation from celestial bodies which reaches the observer. It is possible to determine either the general, integrated intensity of radiation over wide portions of the spectrum, or the distribution of intensity along the spectrum. Indeed, if we know the law of distribution of intensity in the spectrum i.e. if we can represent the intensity i as a continuous function of the wavelength

$$i = \varphi(\lambda).$$

the integrated intensity in the spectral region from λ_1 to λ_2 will evidently be

$$I = \int_{\lambda_1}^{\lambda_2} \varphi(\lambda) d\lambda. \quad (1)$$

The study of the distribution of energy in the spectrum is the problem of spectrophotometry, comparatively new branch of astrophysics. The problem of astrophotometry consists in the determination of the integrated brightness of celestial bodies.

It is assumed in formula (1) that the receiver (which we use in qualitative measurements of radiant energy receives the rays of various wavelengths uniformly. But in reality this is not so, and we should write

$$I = \int_{\lambda_1}^{\lambda_2} F(\lambda) \varphi(\lambda) d\lambda, \quad (1')$$

where the function $F(\lambda)$ determines the spectral sensitivity of the receiver of radiant energy. This spectral sensitivity is quite different for various receivers as well as for various limits of wavelengths λ_1 and λ_2 . Thus the eye is able to receive the radiant energy contained within the limits of $\lambda = 0.4\mu$ and $\lambda = 0.84\mu$. The usual photographic plate is sensitive to rays of wavelength of 0.35μ to 0.5μ . The application of the photoelectric cell enables us to obtain the general intensity in various portions of the spectrum depending on the active substance used in the preparation of the photo-cell. We may therefore divide astrophotometry into three parts corresponding to the visual, photographic, and photo-electric investigations.

However, before we start a detailed exposition of each of the three parts we need to acquaint ourselves with an important phenomenon - the weakening of the light by the terrestrial atmosphere - which influences considerably the results of photometric measurements.

1. Weakening of Light in the Terrestrial Atmosphere.

The terrestrial atmosphere, like any other medium, has the ability to scatter and to absorb radiant energy that reaches it. Therefore, the radiation from celestial objects reaches the observer more or less weakened. This weakening depends on the length of path in the terrestrial atmosphere that a ray has to traverse; the length of the path is a function of the altitude of the celestial body above the horizon or of its zenith distance. It is also clear that weakening of the light will be less the higher the observer is situated above sea level.

The quantity of light lost in the atmosphere is considerable. Thus near the horizon stars appear to be fainter by three magnitudes than at the zenith. Hence the importance of the phenomenon of weakening of light for astrophysics is self evident. We should be able to free the results of the measurements of the brightness of celestial bodies from the influence of the terrestrial atmosphere in order that the brightnesses may be compared to each other. The higher the inner accuracy of the measurements the more careful must we be in the consideration of this phenomenon. It is quite difficult to take due consideration of it from the theoretical point of view; however, at present we may compute it with sufficient accuracy for most practical cases.

The problem is essentially divided into two parts. First, we must know the law of weakening of light depending on the zenith distance of the celestial body, and second, we must determine the quantity of light lost during the passing of the light through a definite thickness of the atmosphere.

§ 1. General Considerations. Let us assume for the present that our atmosphere has the property of diminishing equally the intensity of rays of various wavelengths. Let the ray of intensity I fall upon an infinitely thin layer of an absorbing medium. Let us denote the path of the ray inside the medium by ds . After the ray comes out of the medium its intensity will be $I - dI$ where dI is the loss of light in the layer. We may write the following expression for the loss of light in the layer:

$$dI = -k\rho I ds \quad (2)$$

where ρ is the density of the medium and K is a certain constant depending on the physical properties of the medium.

Let us imagine now that we deal with a layer of air of thickness S and z with the normal to the layer. If we denote the final intensity as I_2 to distinguish it from the initial I , we shall then obtain by integration, on the bases of equation (2), the following relation between I_2 and I :

$$\log \frac{I_2}{I} = -k \int_0^S \rho ds$$

or, taking ρ as constant, we have:

$$I_z = I e^{-k\rho s}.$$

The quantity $e^{-k\rho}$ is called the coefficient of transparency or the coefficient of transmission. Denoting it by c , we have:

$$I_z = I c^s.$$

Let us assume that the terrestrial atmosphere is homogeneous and has a height of l_0 . Then in the case when $z = 0$ we should write for it

$$I_0 = I c^{l_0}$$

or

$$I_0 = I p$$

where $p = c^{l_0}$ is the coefficient of transparency of the terrestrial atmosphere.

In reality the density of the atmosphere diminished as the altitude increases, and therefore the relation between the initial and final intensities of the real atmosphere will be more complex. Namely, we should write

$$I_z = I e^{-k \int \rho ds}, \quad (3)$$

where the integration is carried out along the entire path in the atmosphere. $\int \rho ds$ is evidently nothing else than the total mass of air pierced by the ray of light. Let us denote it by M_z , then

$$\frac{I_z}{I} = e^{-k M_z}.$$

If we denote by I_0 the intensity of the ray when the celestial body is at the zenith, then

$$\frac{I_0}{I} = e^{-k M_0},$$

or taking M_0 as the unit of the air mass, we have

$$\frac{I_z}{I_0} = p^{(M_z - 1)}. \quad (4)$$

Thus, the problem of taking into consideration the influence of the terrestrial atmosphere upon the brightness of the celestial bodies is reduced to the computation of the integral of the exponent in formula (3) or the mass of air M_z . Had we known quite accurately the law of the diminution of density of the air as the altitude increases, we could have solved this problem with rigorous accuracy. But in reality we are compelled to make more or less probable assumptions as to this law, basing ourselves often on experimental data.

Matters stand here very much as in the theory of refraction with which the theory of weakening is closely connected.

There are several theories of weakening. The theories of Lambert and of Bouguer appeared first chronologically. They assume the path of the ray in the atmosphere to be in a straight line. Therefore, these theories satisfy the observations only at comparatively small zenith distances, where the effect of refraction is still not very great. Laplace was the first to regard the path of the ray as following a curve. We shall proceed now to derive his formula for weakening.

§ 2. Laplace's Theory. Let EE' in Figure 32 be the surface of the earth, O be the place of the observer, OZ be the direction toward the zenith, AA' be the boundary of the terrestrial atmosphere. Let us imagine the terrestrial atmosphere to consist of an infinite number of thin concentric spherical layers. Let PP' be one of such layers of density ρ . Let SO be the path of a ray through the atmosphere. The path will be somewhat curved because of refraction. Let us denote by I_2 the intensity of the ray which falls on the layer PP' . Then, according to what was said previously, we have

$$\frac{dI_2}{I_2} = -k\rho ds$$

where ds is an element of the path passing through by the ray. If the radius of the layer is r and the thickness is dr , then

$$ds = \frac{dr}{\cos i}$$

where i is the angle of incidence of the ray upon the layer.

Hence

$$\frac{dI_2}{I_2} = -k\rho \frac{dr}{\cos i} \quad (5)$$

It is known from the theory of refraction that the differential of the refraction (for the layer) of index μ , may be represented as follows:

$$d\zeta = -\operatorname{tgi} \frac{d\mu}{\mu} \quad (6)$$

Besides it is known that

$$\mu r \sin i = \mu_0 a \sin z', \quad (6')$$

where μ_0 is the index of refraction of the layer closest to the terrestrial surface, a is the radius of the Earth, and z' is the apparent zenith distance. Laplace uses these two relations, for the derivation of the correction between weakening and refraction. For this purpose it is also necessary to know the interrelation between the density and the index of refraction of the air. Laplace accepts

$$\mu^2 - 1 = c\rho.$$

Differentiating, we have

$$\frac{d\mu}{\mu} = \frac{cd\rho}{2\mu^2},$$

and therefore the formula (6) may be rewritten as follows:

$$d\zeta = -\frac{c}{2\mu^2} \operatorname{tgi} d\rho.$$

But according to (6')

$$\sin i = \frac{a}{r} \frac{\mu_0}{\mu} \sin z,$$

therefore

$$d\zeta = \frac{ca\mu}{2r\mu^3} \frac{\sin z'}{\cos i} d\rho.$$

Let us transform the right hand part of this equation by substituting other quantities for d . Neglecting the decrease of temperature as the altitude increases, we may write the following on the bases of the Boyle-Mariotte Law:

$$\frac{B}{B_0} = \frac{\rho}{\rho_0},$$

where B and B_0 are the atmospheric pressures at a certain altitude and at the surface of the earth, and ρ and ρ_0 are the corresponding densities.

Hence,

$$\frac{dB}{B_0} = \frac{d\rho}{\rho_0}. \quad (7)$$

The law of the change of pressure with the altitude has the following form:

$$dB = -g\rho dr$$

or, introducing g_0 , acceleration of gravity at the surface of the earth,

$$dB = -g_0 \left(\frac{a}{r}\right)^2 \rho dr.$$

Next, for B_0 we may write the following expression:

$$B_0 = g_0 \rho_0 l_0,$$

where l_0 is evidently, the height of the fictitious homogeneous atmosphere having the density ρ_0 . It is easy to compute l_0 for the mean values of B_0 , ρ_0 , g_0 for the latitude 45° :

$$l_0 = 7.990 \text{ km}$$

It is evidently possible to neglect the changes in the gravitational force for this value.

Thus,

$$\frac{dB}{B_0} = -\left(\frac{a}{r}\right)^2 \frac{\rho}{\rho_0 l_0} dr. \quad (7')$$

From (7) and (7') we obtain

$$dp = -\left(\frac{a}{r}\right)^2 \frac{\rho}{l_0} dr$$

and, finally, we rewrite the differential of refraction in the following form:

$$d\zeta = \frac{c\mu}{2l_0\mu^3} \left(\frac{a}{r}\right)^3 \sin z' \frac{\rho dr}{\cos i},$$

whence

$$\rho \frac{dr}{\cos i} = \frac{2l_0}{c\mu} \frac{\mu^3}{\left(\frac{a}{r}\right)^3} \frac{d\zeta}{\sin z}$$

The factor $\mu^3/\left(\frac{a}{r}\right)^3$ is quite close to unity. Indeed, for a comparatively small extent of the terrestrial atmosphere r differs little from a and the index of refraction μ differs little from unity. At the border of the atmosphere $\mu = 1$, at the surface of the earth $\mu = 1.000294$. Therefore, assuming $\mu^3/\left(\frac{a}{r}\right)^3 = 1$ and substituting the expression for $\rho \frac{dr}{\cos i}$ into formula (5) we obtain

$$\frac{dI_z}{I_z} = -\frac{K}{\sin z} d\zeta, \quad (8)$$

where by K is denoted the constant

$$K = \frac{2kl_0}{c\mu_0}.$$

The expression (8) is called Laplace's formula of the weakening.

Integrating it with respect to the entire path of the ray of light from the border of the atmosphere to the surface of the earth, we obtain

$$\log \frac{I_z}{I} = -K \frac{R}{\sin z'},$$

where R is the total refraction, and I is the intensity of the ray at the border of the atmosphere. We can easily find the refraction R from refraction tables, and the expression for the refraction is as follows:

$$R = \alpha_z \cdot \operatorname{tg} z'.$$

Hence

$$\log \frac{I_z}{I} = -K \alpha_z \cdot \sec z'. \quad (9)$$

Instead of I it is more convenient for us to use the easily determined intensity I_0 at the Zenith. At the zenith

$$\alpha_z = \alpha_0$$

so we have

$$\log \frac{I_0}{I} = -K \alpha_0. \quad (10)$$

By definition $\frac{I_0}{I}$ is nothing but the coefficient of transparency p of the atmosphere. Keeping this in mind, we obtain the reduction to the zenith from (9) and (10):

$$\log \frac{I_0}{I} = -\log p \left[\frac{\alpha_z}{\alpha_0} \sec z - 1 \right] \quad (11)$$

or, in stellar magnitudes:

$$m_z - m_0 = -\frac{\log p}{.4} \left[\frac{\alpha_z}{\alpha_0} \sec z - 1 \right].$$

§ 3. The Experimental Determination of the Weakening of Light. The most extensive series of astrophotometric observations for the determination of the coefficient of transparency of the terrestrial atmosphere in visual rays were made by Muller and Kempf. These determinations were made at three stations which differ in altitude above sea level. When we reduce these determinations to sea level keeping in mind that, according to Laplace's theory, p changes proportionately to the mean pressure of the air, then we shall obtain the following values:

After a critical examination of the determinations of p , Muller came to the conclusion that the most probable value is

$$p = 0.835$$

Hence it is easy to derive that the loss of light by the stars at the zenith is 0^m.20.

On the bases of the observations mentioned above Muller composed tables (see Table I) for Potsdam and Sentis which are used for the reduction of the observed brightness to the zenith. Muller's tables are purely empirical, i.e. they give us the weakening as a function of the zenith distance, as they appear from observations. It is of interest to know how well they agree with those computed on the bases of Laplace's theory, taking $p = 0.835$. It appears, that up to a zenith distance of 80°. Muller's values for Potsdam practically coincide with those computed by Laplace's theory. After 80° there appear some deviations which are small but they show a systematic character. This fact may be due to the following two causes: either the observations of stars at large zenith distances are distorted by a systematic error, or Laplace's theory is not sufficiently rigorous. The first assumption is quite probable because Muller's observations at Potsdam at large zenith distances were made only during exceptionally high transparency whereas the selection of nights for comparatively small zenith distances was not made as carefully. Thus the Potsdam determinations are not quite homogeneous.

On the other hand, Laplace's theory cannot be considered as entirely faultless. Indeed, in the derivation of the formula for weakening numerous arbitrary assumptions were made and certain small factors neglected which may, to some extent, influence the final result. For instance, Laplace assumed the atmosphere to be isotheric which we know is not true.

Station	Altitude in Meters	p
Potsdam	100	0.833
Sentis	2504	0.842
Etna	2942	0.835

§ 4. Bemporad's Work. After Laplace, the most extensive studies in the theory of weakening were made by Bemporad. They are published in several memoirs.

Comparing formulae (4) and (11) we conclude that the mass of air in which the ray of light loses a certain part of its intensity, is expressed as follows according to Laplace's theory:

$$M_z = \frac{\alpha_z}{\alpha_0} \sec Z.$$

Bemporad has shown that M_z , as well as the corresponding weakening at zenith distances exceeding 80° is noticeably different from the computations according to Laplace's theory when we take into consideration the change of temperature in the air as the altitude increases. Taking the temperature gradient as 6° per 1 km, we obtain, at the zenith distance of 87° , a difference in the weakening of $0^m.1$ as compared with that computed according to Laplace. This difference increases as the temperature gradient increases. Bemporad has developed a more rigorous theory of the weakening. We shall deal here chiefly with its final results.

In its final form Bemporad's expression for the mass of air has the following form.

$$M_z = F(T)\sqrt{T} \sec Z + F_1(T)[\sqrt{T} \sec Z]^3 + F_2(T)[\sqrt{T} \sec Z]^5 + \dots$$

where

$$T = \frac{1}{1 + \frac{1}{2}\gamma \cot^2 Z}, \text{ and } F_i(T) = \varphi_i(T) + \psi_i(T),$$

and

$$\gamma = \frac{m\beta R}{1 + mt},$$

where m is the coefficient of the expansion of gases under the influence of heat,

β - is the temperature gradient of the atmosphere.

Bemporad developed extensive tables of M_z , computed by him on the bases of the formula above with respect to argument z . These tables refer to 45° of latitude, 0° temperature, and a pressure of 760 mm, and they are computed for altitude up to 3000m above sea level. In the following Table 2 are given the masses of the air computed by Laplace's and by Bemporad's methods and the differences of these computations. The table begins only with $z = 80^\circ$, as for smaller zenith distances the two theories gives identical results. In these calculations, $p = 0.835$, and the air mass pierced by the ray when the star is at zenith is taken as unity.

Table 2

Zenith Distance	Air Masses		The Difference in the Weakening Laplace - Bemporad
	Laplace	Bemporad	
80°	5.563	5.600	- 0 ^m .01
82	6.818	6.884	- 0 .01
84	8.768	8.900	- 0 .03
86	12.125	12.439	- 0 .06
87	14.835	15.364	- 0 .10
88	18.835	19.787	- 0 .19

The weakening of light at a given place of the terrestrial surface depends also on the pressure and temperature of the closest layer of air. Thus, these meteorologic elements also need to take into consideration when an accurate record of the influence of the atmosphere on the observed brightness of the celestial body is taken. Their influence also shows itself upon the change of magnitude M_z as well as in the change of the coefficient of transparency p . Bemporad gave a little table by which these influences may be taken into consideration.

§ 5. Dependence of the Weakening on the Wavelength. We had previously assumed that the atmosphere weakens equally the rays of various wavelengths.

In reality, the atmosphere has the property of selective weakening of light, and the weakening is the greater, the smaller the wavelength. Such a property of our atmosphere becomes understandable when we recall that the causes of the weakening in the atmosphere are: the thermal absorption of energy by permanent gases of which the atmosphere consists, the scattering of light in the thermal absorption by the larger particles in the atmosphere, and molecular diffusion. Rayleigh gave the following formula for the dependence of the coefficient of transparency upon the wavelength (in the assumption of molecular diffusion):

$$\log_e p_\lambda = -\frac{K}{\lambda^4}$$

where K is a constant. According to L. King, the exponent K in the expression of the coefficient of transparency

$$p = e^{-kp}$$

may be represented as follows:

$$k = \frac{32}{3} \left[\pi^3 (n_0 - 1) \frac{l_0}{N_0} \lambda^{-4} + \alpha_0 l_0 \right] \frac{B}{B_0} + Q,$$

where n_0 , N_0 , B_0 are respectively the index of refraction of the air, the number of molecules in one cubic centimeter of air, and the pressure at sea level; l_0 is the altitude of the fictitious homogeneous atmosphere of density, B is the atmospheric pressure at the place of observation. The term $\alpha_0 l_0$ expresses the energy which is turned into heat during the passing of the light through the atmosphere. Finally, Q represents the influence of the atmospheric dust.

The phenomenon of the selectivity of the weakening by the terrestrial atmosphere, aside from its theoretical significance, is also very important for practical astrophotometry. Indeed, as the receivers of light (the eye, the photographic plate, the photoelement), used in the measurement of stellar brightness, are not of the same sensitivity, it is evident that in the reduction of the measurements it is necessary to use different coefficients of transparency. Even the measurements made by the same method require, in their reductions for the atmospheric weakening, the knowledge of the dependence of the coefficient of transparency p upon the color of the measured stars.

It was said above that the value 0.835 for p is sufficiently good for the reduction of the usual visual observations. However, this number differs depending on the color or the spectral type of stars. Indeed, Zipler obtained from his observations at the observatory of Babelsberg, near Berlin, the following results:

2. Visual Photometry

§ 6. The Unit of Stellar Brightness. The Normal System of Stellar Magnitudes.

Visual photometry is perhaps the oldest branch of astrophysics, for even the ancient astronomers paid attention to the difference in brightness of the celestial bodies. Astrophotometry began when the first attempt to classify the stars according to their brightness was made. As far as we know from written documents, the first such attempt was recorded by Ptolemy in 138 A.D. Ptolemy divided 1022 stars, observed by the unaided eye, into 6 classes or magnitudes according to their brightness. The brightest stars were referred to the first magnitude, and the faintest to the sixth. Such a subdivision was used in later lists of stars observed by the unaided eye. After the invention of the telescope, this scale was continued for fainter stars, i.e. for telescopic stars. The name stellar magnitude is used for the present day as a unit of stellar brightness.

The historically developed classification of stellar brightness may seem to be at first sight, entirely arbitrary. But in reality it is based upon Wev Weber's psycho-physical law which establishes the quantitative relation between the magnitude of irritation of our sense organs and the corresponding sensations. Fechner has formulated this law with respect to the human eye.

Let us denote by dE the increase of the power of sensation corresponding to the increase dI of brightness of the light source. Then, according to Fechner:

$$dE = c \frac{dI}{I}.$$

Integrating it we obtain:

$$E = c \log I + C.$$

For another pair values E_0 and I_0 , corresponding to each other, we shall similarly find:

$$E_0 = c \log I_0 + C,$$

Hence

$$E - E_0 = c \log \frac{I}{I_0}.$$

This may be rewritten as follows:

$$\frac{I}{I_0} = e^{\frac{E - E_0}{c}}.$$

Hence, we obtain the following formulation of the Weber-Fechner Law: if the brightnesses change according to a geometrical progression, the corresponding sensations change according to an arithmetical progression.

Let us now deal with two stars one being one magnitude brighter than the other: one of magnitude m , the other of magnitude $m + 1$. If we denote their brightness by I_m and I_{m+1} , then, according to the foregoing example

$$E_m = c \log I_m + C$$

$$E_{m+1} = c \log I_{m+1} + C,$$

whence

$$E_{m+1} - E_m = c \log \frac{I_{m+1}}{I_m}.$$

Thus, if the accepted division of stars into classes has as its basis the Weber-Fechner Law, then

$$c \log \frac{I_{m+1}}{I_m} = \text{const.} = k.$$

Denoting $\frac{k}{c}$ by $\log 1/\rho$ we have

$$\frac{I_m}{I_{m+1}} = \rho.$$

Numerous investigators tried to determine the magnitude ρ and it appeared that ρ is almost always close to 2.5. According to Muller, for bright stars the logarithm of ρ (logarithms whose base is 10) is as follows:

$$\log \rho = 0.340$$

For faint stars

$$\log \rho = 0.394$$

The difference for ρ between the bright and the faint stars may be explained, in part, by the systematic errors in visual evaluations of stars as well as in photometric determinations of the ratio of their brightness.

At present, the normal scale of stellar magnitudes is the one in which, according to Pogson's proposal,

$$\log \frac{I_m}{I_{m+1}} = 0.4$$

i.e. the ratio of the stellar brightness of two consecutive magnitudes is constant and is equal to 2.512

Thus if we take for instance, the brightness of the sixth magnitude as unity then the brightness of the fifth magnitude will be 2.512, the brightness of the fourth magnitude will be $(2.512)^2 = 6.25$, and so forth. In general, the ratio of the brightness of stars of magnitude m and n is expressed as follows:

$$\frac{I_m}{I_n} = 2.512^{n-m}.$$

Conversely, the difference of magnitudes of two stars is expressed by their

brightnesses as follows (the logarithms are to the base ten):

$$m_2 - m_1 = \frac{\log I_1 - \log I_2}{0.4} = -2.5(\log I_2 - \log I_1).$$

This formula is called Pogson's formula.

The position of the zero point of the system of stellar magnitudes, or the origin, is less clear. At present, the zero point of the H.R.P. (Harvard Revised Photometry) catalogue, published by Pickering in the 50th volume of the Annals of the Harvard College Observatory, is taken as the normal zero point. It has been so selected that the photometric magnitudes of 100 stars, close to the polar star, from the 2nd to the 6th magnitude agree, on the average, with the data taken from the most complete source of visual evaluations of stellar magnitudes, namely, the Bonner Durchmusterung (or B.D) of the northern sky.

From the point of view of the determination of the zero point of stellar magnitudes, it is of interest to examine the expression for the brightness of celestial objects in units accepted in Physics. If we take as the unit of illumination the lumen, i.e. the illumination caused by Hefner's candle at a distance of one meter, it appears that a star of zero magnitude causes an illumination of 2.1×10^{-6} lumens. Hence it follows that Hefner's candle, at a distance of 1 kilometer corresponds in brightness to a star of the first magnitude. The illumination of the Sun when it is at the zenith is equal to 1.5×10^5 lumens, and that of the Moon is 0.3 lumens.

§ 7. The Photometric Properties of the Eye. Before we go into the description of the methods and apparatus used in visual photometry for the measurement of the brightness of stars and other celestial bodies, it is necessary to consider the properties of the human eye which is, in this case, the receptor of the radiative energy.

The mechanism of sight consists, as is known, essentially in that the rays of light, passing from the object under consideration through the opening of the pupil of the eye, forms a real image of the object on the retina which covers the inner surface of the eye. In this process the crystalline lenses is very important. It is a double convex lens which is able to change the curvature of its surfaces. The retina consists of a tremendous number (about 140,000, 000) of elements sensitive to light, the so called cones and rods, which are connected with the fibres of the optic nerve, the excitation of which is perceived by us as light. And, as was said previously, this sensation is connected with the corresponding excitation of the optic nerve by a logarithmic relation.

The cones and rods fulfill in the process of sight two somewhat different functions. The cones are excited only under the action of considerable illumination whereas the rods begin to act with very small illumination to which the cones do not react. The rods are suited for "twilight" sight.

The opening of the pupil of the eye has the property of extension and contraction depending on the degree of illumination. Its diameter in complete darkness, or very faint light, reaches about 8mm. This quantity is of great significance in practical astrophotometry: evidently, the exit aperture of the photometric instruments must not exceed the diameter of the opening of the pupil of the eye, for otherwise the entire light collected by the instrument will not reach the eye fully.

The eye has a so called threshold of sensitivity: in order to obtain a noticeable excitation of the optic nerve a certain quantity of radiant energy is required. It has been established by special experiments of the Lick Observatory that under favorable conditions (complete darkness, a completely dark background) stars of 8.5 magnitude, may be noticed. Using these experiments as a basis, H. N. Russell computed the minimum amount of radiant energy which is capable of exciting the sensation of light. It is equal to 7.7×10^{-10} erg/sec.

Of great importance for photometry is the ability of the eye to notice the difference in brightness of the objects under examination. As appears from under average illumination, as small a difference in brightness as 1.7%. At the extremely high or low illuminations increases or decreases, this ability of the eye rapidly diminishes.

The sensitivity of the eye towards radiation differs for various wavelengths. In this case, as well as in the cases above, the eye of different individuals differ greatly. Nevertheless, it is possible to obtain from numerous investigations the mean values which characterize the spectral sensitivity of the retina. It appears that beginning with $\lambda = 400\mu$ the sensitivity of the retina increases continuously up to $\lambda = 555\mu$ where it reaches a maximum and then decreases to zero at 760μ . This is true for comparatively great illumination, for very small illumination the maximum for sensitivity shifts toward the shorter wavelengths.

The dependence of the color sensitivity of the eye upon the brightness is the cause of the so called Purkinje phenomenon, which is of great importance in astrophotometry and especially in spectrophotometry. This phenomenon will become clearer from the following description. Let us compare two sources of light, two stars for instance, one of them having a bluish, the other a reddish color. Let them have the appearance of equal brightness. If we now weaken the two sources of light equally, we shall notice that the reddish source of light will decrease in brightness faster than the bluish one, and their equality in brightness will disappear. Conversely, if the brightness is increased, the reddish source of light will appear to us to be brighter as compared with the bluish source. Therefore, in the more accurate astrophotographic investigations we must compare only stars of the same color. A more detailed examination of the Purkinje phenomenon will be given in the chapter on spectrophotometry.

The relative number of cones and rods is not the same at different parts of the retina. The rods become more prevalent as we move from the center of the edges. Thereby is explained the phenomenon of peripheral vision at which fainter objects may be observed than those by direct vision. In photometric observations it is necessary to keep the image of each of the stars under comparison at the same place on the retina.

§ 8. The Evaluations of the Brightness of Celestial Bodies by means of the Eye without the Aid of Photometric Instruments. Our eye is, of course, not a photometric measuring instrument. It can only evaluate the ratios of brightnesses of objects under comparison within certain limits with a small degree of accuracy. Nevertheless, direct evaluations of stellar magnitudes, which alone were possible in the past, are in use even at the present time quite extensively. This is done, for instance in the observations for stellar catalogues by meridian instruments and in the observations of variable stars, minor planets, and miscellaneous objects. The method depends upon the ability of the eye to remember the force of the excitation of light caused by stars of different brightness. A long training of the eye with regard to stars of definite brightness leads finally to a development of a certain mental scale of stellar magnitudes. Such a scale may be developed by a given observer for a very wide interval of brightnesses. However, it is quite clear that this method of evaluation of stellar magnitudes is not very accurate because the eye is compelled to evaluate the absolute illumination caused by the star at the retina. But this illumination depends greatly upon the condition of the transparency of the atmosphere; besides, the retina may not always remain unchanged from day to day. As an example, Table 3 shows the average errors of stellar magnitudes of certain old catalogues given by Pickering.

In the newer catalogues the accuracy is considerably higher. This improvement was achieved by a more careful reduction of observations as well as by taking precautions which protect the eye from outside irritations. For instance, the accuracy of the stellar magnitudes given by the Bonner Durchmusterung (B.D) characterized, according to Pickering by a mean error of only $\pm 0^m.24$.

A still greater accuracy is obtained in modern times in the evaluation of stellar magnitudes made during their observation by meridian instruments. Because of the application of light absorbing screens set in front of the objective of the tube, the observer evaluates the brightness of stars only in a narrow interval of the mental scale, of the order of two stellar magnitudes. A careful training of the eye and the reduction of the observations for the change in the transparency of the atmosphere enable us to obtain a greater accuracy. Thus, for instance, the Pulkovo Catalogue of positions of 3397 stars in the zone between $+ 39^\circ$ and $+ 46^\circ$ gives us stellar magnitudes with a mean error of $\pm 0^m.14$ according to M. Morin.

The Step Method. A different method from the one expounded above is the method of steps which consists in the evaluation of the difference in

brightness of two stars. This method was first applied by W. Herschel at the end of the eighteenth century and was subsequently developed by Argelander. The classical work of Argelander on the study of the change in brightness of variable stars (this work was done by the method of steps in the middle of the nineteenth century) has rendered this method widespread in astronomical practice, especially in the investigation of variable stars. This method is also known as Argelander's method.

Table 3

Magnitudes	Ptolomy	Tycho Brahe	Hevelius
2	$\pm 0^m.60$	$\pm 0^m.66$	$\pm 0^m.52$
3	0 .47	0 .55	0 .52
4	0 .44	0 .51	0 .48
5		0 .43	0 .37
6			0 .47

In order to determine the magnitude of a star we select close to it one or more comparison stars which differ as little as possible from the star under investigation. Let us fix the eye upon the star "a" under investigation, and then let us quickly turn the eye upon star "b". Repeating this process several times we finally evaluate the difference of the stellar magnitudes in the two stars. If the stars appear to be equally bright, we record the results of the comparison as follows:

ab or ba

If at first sight the stars appear to be of the same brightness, but upon repeated comparisons one of them will always (except in rare cases) appear brighter than the other, we then accept the smallest difference that we noticed as one step and the result of comparison is recorded in the following form:

alb.

where the brighter star appears first.

If the star "a" appears to be indubitably brighter than "b" but the difference in brightness is small, we record:

a2b.

If the difference between the two stars can be perceived immediately, we shall record the result as follows:

a3b.

Greater differences in the brightness may be evaluated in four or more steps. If the difference exceeds four steps, it is then recommended to select another comparison star.

The magnitude of the step is quite different for various observers and it is subject to various fluctuations besides. Therefore, it must be determined from the observations themselves. For this purpose it is necessary to have at least two comparison stars the magnitudes of which are well known. Let the result of comparison of star "a" with the comparison stars "b" and "c" be recorded as follows:

bpa and aqc

where p and q are the number of steps. If the magnitudes of the comparison stars will be m_b and m_c , it is evident that we may write the following equations for the star "a":

$$m_a = m_b + p \cdot s$$

$$m_a = m_c - q \cdot s,$$

where s is one step expressed in stellar magnitudes. Subtracting one from the other we obtain

$$s = \frac{m_c - m_b}{p + q}.$$

The magnitude of one step for an observer of little experience is of the order $0^m.1$, which diminishes as his eye becomes trained. For certain well trained observers this quantity is reduced to $0^m.3$. However, even well trained observers show large jumps on both sides. This circumstance is one of the greatest deficiencies in Argelander's method, because the amount of the step must be always kept under control, and this is possible only in large series of observations.

A development of Argelander's method is that of Pickering where the mental scale is of little importance. It is a purely interpolational method. Two comparison stars are selected, one which is brighter than the star under consideration, and another which is fainter. The difference in brightness between the star under consideration and the comparison stars is evaluated in one tenth parts of the interval $m_b - m_c$. The result is recorded as follows:

bpage

Since

$$p + q = 10,$$

the equation for the star "a" will be

$$m_a = m_b - \frac{m_c - m_b}{10} p = m_c + \frac{m_c - m_b}{10} q.$$

Thus, in Pickering's method the step is entirely absent as a certain smallest difference of brightnesses which may be noticed by the observer's eye.

Both methods are not free from certain systematic errors the chief of which are as follows:

a) "the interval error". Most observers evaluate the different intervals of brightness by numbers of steps which do not correspond to the true ratio of the brightness. Therefore, the size of the step depends upon the size of the interval from which it is derived:

b) "the error of interpolation". The essence of this error is that in Pickering's method, where the magnitudes of two unequal intervals of brightness are evaluated these evaluations depends upon the ratios of the intervals themselves;

c) "the decimal error", which is a purely psychological error of the Pickering method. Some observers prefer certain evaluations to others.

d) "the error of position". It is equally dangerous in Argelander's and Pickering's method. The result of the comparison depends on the mutual position of the stars compared. Two kinds of error of this type are distinguished: one depends on the distance between the stars, the other depends on the position of the line, joining the stars, relative to the line joining the observer's eyes;

e) "the error of background". The evaluations depend upon the brightness of the background of the sky at the moment of observation: the results of observation on moonlight nights and on moonless nights are noticeably different;

f) "the error of color". For purposes of comparison it is necessary to select stars of the same color as much as possible, otherwise there is the danger of an error of the Purkinje phenomenon.

The accuracy of the results obtained by the methods just expounded is superior to the plain evaluations of the unaided eye. When good care is taken to avoid or to lessen the influence of systematic errors, the mean error of a single determination of brightness for experienced observers is of the order of $\pm 0^m.08$.

§ 9. Principles of Instrumental Photometry. A further increase in the accuracy of astrophotometric determinations is obtained by means of special instruments, the so called astrophotometers, the use of which relegates to the eye the mere judgment of the equality of brightness. There are two basic types of astrophotometers: 1) those that operate on the principle of extinguishing the brightness, 2) and those which work on the principle of equalizing the brightness.

Let us consider the first case. Let us have two stars of brightness i_1 and i_2 . Let us weaken by means of certain devices their brightness in such a manner that the stars become entirely invisible, i.e. in such a way that their brightness is at the threshold of sensitivity of the eye. Let us, for this purpose, weaken the first star in the ratio of s_1 , and the second star in the ratio of s_2 . Then, if the conditions of observation are the same for both, we shall have:

$$i_1 s_1 = i_2 s_2,$$

whence

$$\frac{i_1}{i_2} = \frac{s_2}{s_1}$$

and the difference in stellar magnitudes is

$$\Delta m = -2.5 \log \frac{s_2}{s_1}.$$

In the second case we make the brighter star become fainter until it is equal in brightness to the fainter star. Let $i_1 > i_2$ and in order to reach a photometric equality it is necessary to make the first star fainter in the ratio s . Then

$$i_1 s = i_2$$

and

$$\Delta m = -2.5 \log s.$$

In order to realize these principles of measuring the difference in brightness, it is evidently necessary to have at our disposal means which enable us to make fainter continuously the brightness of the object under consideration

and to measure accurately the amount of fading. Usually, the following methods are used for this purpose:

- a) the diaphragming of the objective of the instrument;
- b) the diminution of the light by an absorbing medium;
- c) the application of the properties of polarized light.

a) Diaphragms. This is the simplest method of decreasing the brightness of the focal image of stars. If we denote by σ the area of the aperture of the diaphragm which is placed in front of the objective, the brightness of the focal image will be proportional to σ and the change in brightness is proportional to the change of the area of the aperture;

$$dI = c d\sigma.$$

The forms of the diaphragms used in photometry are various. The simplest diaphragms -- the round ones -- are shaped in the form of the so called "iris" diaphragms in order to obtain a continuous change in brightness.

The quadrilateral diaphragm ("The cat's eye") is formed by the opaque plates having cuts of the form of the letter V, which may slide one along the other (see Figure 33), and their shift may be measured by a scale. The reading gives us the length of the diagonal of the quadrilateral aperture of the diaphragm. Let the largest extension of the plate (and therefore of the diagonal) take place at the reading s_0 ; then at a reading s the focal image will be weakened in the ratio of s^2/s_0^2 .

The sector diaphragm is so constructed that it may leave only a part of the objective free and this part has the form of a sector. The angle of the sector may change continuously. The change in brightness of the image given by the objective is proportional to the change of the angle of the sector. Such a diaphragm is usually brought into rapid motion in order to avoid distortions of the images.

The application of diaphragms in photometry is not free from numerous objections. As is known, the focal image of a star is represented as a disk the diameter of which depends on the diameter of the free aperture of the objective. Therefore changing the aperture of the diaphragm changes the size of the diffraction disk. Because of this the law of proportional areas of the aperture will not hold when the diaphragm aperture is very small.

Besides, it is evident that having diaphragmed the objective we shall obtain images formed by different parts of it. Finally, together with the change in the brightness of the image the brightness of the background will also change which circumstance may lead to serious systematic errors.

b) The Photometric Wedge. One of the best methods of producing a continuous change in brightness used in astrophotometry is the application of the

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photometric wedge. It consists of two glass prisms having a very small angle of refraction. One of the prisms is made of the usual transparent glass, the other is made of smoky glass which absorbs light greatly. The two prisms are glued to each other in such a way that they form together a flat parallel plate.

Let on Figure 34, AB be the length of the wedge, and AD = BC be its thickness. Let us denote the length by l and the thickness by d . The shaded part represents the prism made of smoky glass. Let two rays of light fall on the wedge at the points E and F. Inside of it both rays pass the distance a and e through the smoky glass and distance b and f through the clear glass. Let the respective brightness of the rays be I_1 and I_2 . As is known, if the ray of light passes through the distance s in a homogeneous absorbing medium, its intensity i_1 , after it comes out from the medium, is connected with the initial intensity i_0 by the following relation

$$i_1 : i_0 = \alpha^s,$$

where α is the coefficient of transparency of the medium. Hence, for our case the relation will be:

$$I'_1 : I_1 = k^a c^b \quad \text{and} \quad I'_2 : I_2 = k^e c^f$$

Here I'_1 and I'_2 are the intensities of the two rays after they come out from the wedge, and k and c are the coefficients of transparency of its two halves. Putting the foregoing expressions into logarithmic form and keeping in mind that $b = d - a$ we obtain

$$\left. \begin{aligned} \log I'_1 - \log I_1 &= a(\log k - \log c) + d \log c \\ \log I'_2 - \log I_2 &= e(\log k - \log c) + d \log c \end{aligned} \right\} \quad (12)$$

Let us assume that at two positions of the wedge we observe the weakening of both rays. Then $I'_1 = I'_2$ and hence for this case we shall have from (12):

$$\log I_1 - \log I_2 = (e - a)(\log k - \log c).$$

If we denote the section EF by n , then

$$e - a = \frac{nd}{l}$$

and therefore

$$\log I_1 - \log I_2 = \frac{nd}{l} (\log k - \log c).$$

Converting into the stellar magnitudes we find:

$$-2.5 (\log I_1 - \log I_2) = -2.5 \frac{d}{l} (\log k - \log c) n$$

or, denoting

$$\beta = -2.5 \frac{d}{l} (\log k - \log c), \quad (12')$$

we obtain

$$\Delta m = \beta \cdot n$$

The quantity β depends only upon the properties of the glasses of which the wedge is made and upon the angle of the prisms. It is called the constant of the wedge. This constant represents the change in brightness of the source of light observed through the wedge when the latter shifts by a unit of length. If the coefficients of transparency of the glasses is well known, it is easy to compute the constant of the wedge by formula (12'). However, it may be determined still better from the observations made with the wedge. Such a determination may be made from the observations of already well determined stars or from the measurements of an artificial source of light the brightness of which may be changed in a definite ratio. One of the best methods for this purpose the photometric collimator proposed by Tserassky. This device consists of a toothed disk placed between the source of light and the wedge under investigation. The disk is brought into rapid rotation and it is possible to change the openings between the teeth, and thus to change the brightness of the source of light in a known proportion.

Such an investigation should be made for various points of the wedge in order to convince ourselves that β has the same value for the entire length of the wedge. When the glass of the wedge is not sufficiently homogenous or when the surface of the two prisms has not been sufficiently well polished, may not have the same value for the entire length of the wedge. Whenever such a situation obtains, it is necessary to construct a curve of the absorption of the wedge and use it for the reductions of the observations.

Sometimes, instead of a glass wedge a photographic wedge is used which is a strip of an ordinary photographic plate which has been subjected to uniformly changing illumination. After its development it appears blackened, and this blackness or non-transparency will uniformly increase towards one of the ends. The method of preparation of which such wedges has been developed by E. King.

A suitable wedge constant should be selected for good photometric work. For the usual wedges used in astrophotometry the most advantageous value of is contained from 0.15 to 0.20 of a stellar magnitude per mm of the length of the wedge. A good carefully developed wedge enables us to work through an interval of 4 -5 stellar magnitudes.

An important property of the wedge is its neutrality. It should weaken the rays of various wavelengths equally.

c) Polarization Prisms. The method of continuous change of the brightness of stars by means of the application of the properties of polarized light is extensively used in astrophotometry. The light of a star is made to pass through polarizing prisms. The Nicol prism is used most often; it consists of two halves of an Iceland spar crystal which are glued together by Canadian balsam; a section of it appears in Figure 35. The plane of the drawing is the

plane of the main cross section of the crystal which is cut along the plane a b perpendicularly to the plane of the main section. The ray E F, falling in the direction of the long side of the prism is divided into two rays, the ordinary and the extraordinary rays, which have almost the same intensity. The ordinary ray is reflected from the cross sectional plane and turns aside. The extraordinary passes through the prism becoming polarized in the plane which is perpendicular to the main section of the crystal. In other polarization prisms (Rochon, Wallaston) the light comes out in the form of two diverging pencils of light, the ordinary and the extraordinary one.

If the polarized light is made to pass through a Nicol the intensity of the ordinary and the extraordinary rays will change depending on the angle between the main cross sections of the polarizer and the analyser. This change takes place according to the Malus Law. Let the initial intensity of the ray be I. After the ray passes through the polarizer, the intensities of the ordinary and extraordinary rays will be:

$$O = \frac{1}{2} m_1 I; \quad E = \frac{1}{2} m_2 I.$$

where m_1 and m_2 are the coefficients of transparency of the crystal for the two rays. They are close to each other. After the rays pass through the analyser their intensity is expressed as follows:

$$O' = \frac{1}{2} m_1^2 I \sin^2 \varphi; \quad E' = \frac{1}{2} m_2^2 I \cos^2 \varphi,$$

where φ is the angle between the main sections of the prism.

§ 10. Wedge Astrophotometers. There are two types of photometers in which the photometric wedge serves the purpose of measuring the quantity of light: the type based on the principle of extinguishing and the type based on the principle of equalization.

The simplest scheme of the photometers of the first type is represented on Figure 36. O is the objective of the refractor, K is the wedge placed near the focus of the refractor, o is the eye piece. The wedge may be shifted in the focal plane by means of a rack and pinion movement, and the length of its shift may be read on a scale fastened to the outer side of the box containing the wedge. At the focus of the objective are placed two narrow metallic plates which serve for the fixing of the plane of the field of view where the compared stars are brought together. The wedge is shifted until the star under observation is about to become extinguished: at this moment the illumination created by the star on the retina is equal to the threshold of its sensitivity. The same operation is made with the second star. If the wedge is entirely homogeneous, the difference in brightness will be: $\Delta m = \beta (S_1 - S_2)$,

where S_1 and S_2 are reading on the scale. If the wedge is not entirely homogeneous, the absorptions corresponding to the reading are taken, and then we have:

$$\Delta m = p_2 - p_1.$$

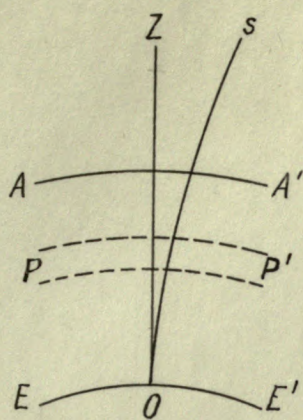


Fig. 32.

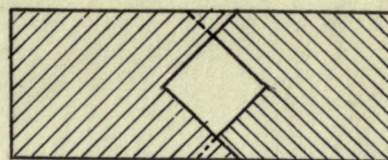


Fig. 33.

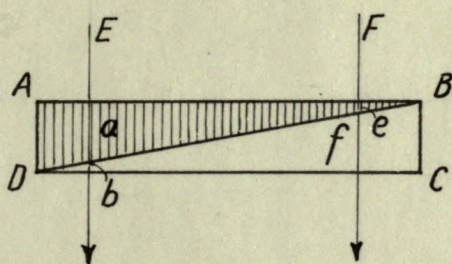


Fig. 34.

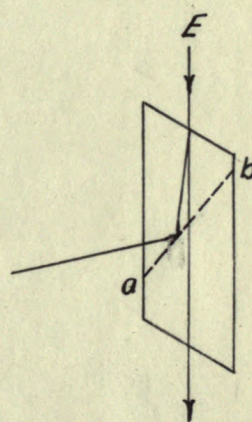


Fig. 35.

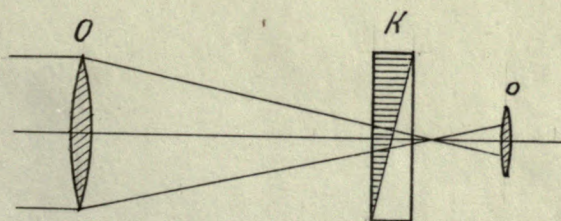


Fig. 36.

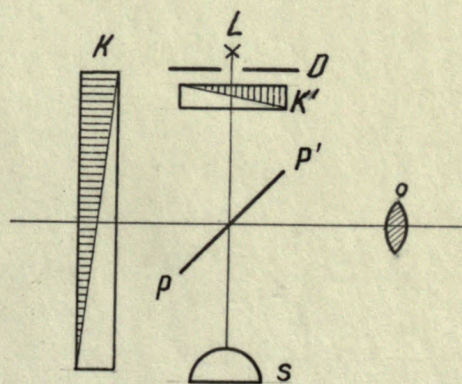


Fig. 38.

A very important condition for successful work in observations by the extinguishing method is a careful protection of the eye from all sorts of outside irritation by light. Otherwise, the threshold of sensitivity of the retina will not be constant. In order that the observer may be released from the necessity of recording the readings of the wedge, Toepfer constructed a registering device in the photometer. (Figure 37). The device consists of a scale with divisions made in raised figures on the frame carrying the wedge. A paper ribbon is pressed to this scale by a special level and a portion of the scale and an index are pressed upon the paper.

Such a device is also useful in many other types of astrophotometers.

Rosenberg's Photometer. By means of a simple additional device Toepfer's wedge photometer may be adapted for the equalization method. For this purpose in place of the eye piece an instrument, the scheme of which is shown in Figure 38, is attached to the telescope.

K is the wedge, L is a small incandescent lamp, PP' is a plane parallel plate set at an angle of 45° to the optical axis, D is a plate with a very small aperture, S is a small silvered hemisphere, o is the eye piece. The rays from the lamp L, passing through the aperture of the diaphragm D, fall upon the hemisphere S and being reflected from the plate PP' form an artificial star at the focus of the eye piece. Its brightness may be adjusted by means of a small wedge K' to the brightness of the faintest star that is measured by wedge K.

Graff's Photometer. This photometer also works according to the principle of equalization, but here the main wedge weakens not the natural star but the artificial star. Its construction is clear from Figure 39. The light of the incandescent lamp L, passing through the lusterless plate S, is concentrated by the lens M_1 on the aperture of the diaphragm D. Its image, formed by the second lens M_2 may be observed by the eye piece o_1 , after the image has been reflected from the plane parallel plate PP', or by the eye piece o_2 through the plate PP'. The wedge K changes the brightness of the natural star. The existence of two eye pieces enables us to widen greatly the interval of the measured brightnesses; this interval may be as large as 10 stellar magnitudes when the wedge is very good. The eye piece o_1 is used for faint stars, the eye piece o_2 is used for bright stars.

Graff's photometry enables us to measure not only point sources of light i.e. stars, but also of bright areas such as those nebulae, planetary surfaces, etc. For this purpose the lens M_2 is moved further away and instead of the plane parallel plate a Lummer Brodhun prism is used. It consists of two rectangular prisms A and B united by the hypotenuse surfaces (see Figure 40). One of them is made convex with the exception of a small part limited by a circle. Both prisms, are closely joined, and therefore on the small area of their

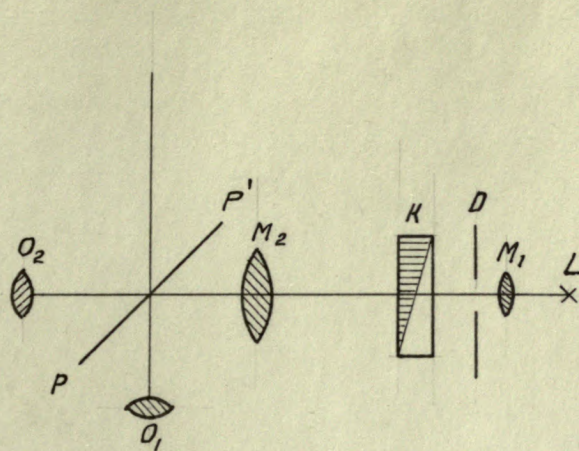


Fig. 39.

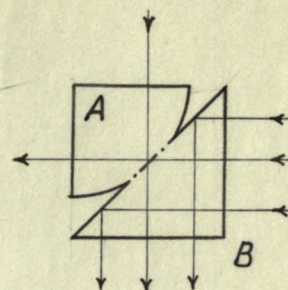


Fig. 40.

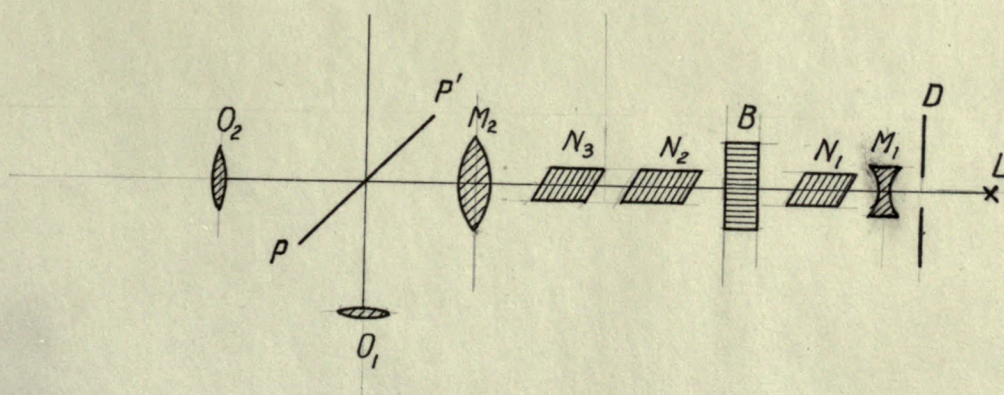


Fig. 41.

tangency no reflection of light takes place. The eye, placed below, will see an elliptic field of tangency in the transmitted light; the surrounding field is formed by the reflected light from the righthand light. The prism is placed in the photometer in such a manner that the rays of the object under investigation pass through the field of tangency. The brightness of the surrounding background is changed by the wedge K until the brightness becomes equal. At this moment the field of tangency ceases to be noticeable on the illuminated field of sight of the eye piece.

The method of equalization is superior to the method of extinguishing in regard to the accuracy of the results. Thus, it appears from a large series of investigations that the difference in brightness of two stars is determined with an average error of no less than $\pm 0^m.1$ for the extinguishing method, whereas the average error for the equalization method may be under favorable circumstances as low as ± 0.05 .

§ 11. The Polarization Astrophotometers. Of a number of astrophotometers based on the properties of polarized light Zellner's photometer is the most widespread. The essence of this contrivance which is added to the eye end of the refractor is clear from Figure 41.

The rays from lamp L pass through the diaphragm D and the concave lens M_1 and they proceed through the Nicol prism N_1 . Next they pass through the quartz crystal B, the two Nicol prisms N_2 and N_3 , and, finally, they form, by means of the collecting lens M_2 an image of the aperture of diaphragm D in the focal plane of the objective after they were reflected from the plane parallel plate PP'. The various apertures of the diaphragm D enable us to equate the diameter of the artificial star to that of the natural star. The diameter may vary depending on the size and quality of the objective of the refractor and the atmospheric conditions. The system of the Nicol prism N_1 and the plate of quartz crystal B represents the colorimeter, i.e., a device based on the phenomenon of chromatic polarization of light by means of which we may change the color of the artificial star by turning the system around its axis. The colors obtained are too sharp and not quite pure; therefore, the colorimeter is not used for measuring the color, but is used merely for the purpose of equating the color of the artificial star to that of the natural one.

The strictly photometric device is the system of Nicol prisms N_2 N_3 . The Nicol N_2 may rotate around the axis while the Nicol N_3 remains stationary; the angle of rotation is read upon the circle of intensities to an accuracy of $0^{\circ}.1$. The brightness of the artificial star will change according to Malus' law. The indicator of the circle is so arranged that for Nicols set for complete weakening the reading is zero. Then the change in brightness will take place according to the following formula:

$$I = c \sin^2 \varphi.$$

In Zellner's photometers of the newest constructions there is a side eye

piece for observations of bright stars, and there is also a device for the registrations of the readings on the circle of intensities on the paper ribbon. Previously, a kerosene burner whose flame remains constant for several hours was used as a source of light. At present incandescent lamps are used for this purpose, and the intensity of the current is controlled by a milliammeter. Figure 42 shows one of the new photometer models made by Toepfer.

The computation of the observations are made by the formula:

$$\Delta m = -5 (\log \sin \varphi_1 - \log \sin \varphi_2), \quad (13)$$

where φ_1 and φ_2 are readings of the circle for two stars, and Δm is the difference of their magnitudes which is to be determined.

The equalization of the brightness of the natural and artificial stars must be made at four positions of the circle of intensities in order that the errors depending on zero point of the circle and on the eccentric position of the Nicol prisms may be eliminated. Let us assume that the following readings were obtained: $\alpha_1, \alpha_2, \alpha_3, \alpha_4$. Let us denote by α_0 the zero point, i.e., that reading of the circle at which the artificial star disappears, and let us denote by ϵ the error of eccentricity. The circle of intensities is usually so divided that the subdivisions go from zero to 90° on both sides and then go from 90° to 0° again. Under these circumstances the correct reading will be expressed as follows:

$$\begin{aligned} \varphi_1 &= \alpha_1 + \epsilon_1 + \alpha_0 & \varphi_3 &= \alpha_3 + \epsilon_3 - \alpha_0 \\ \varphi_2 &= \alpha_2 + \epsilon_2 + \alpha_0 & \varphi_4 &= \alpha_4 + \epsilon_4 + \alpha_0. \end{aligned}$$

Taking into consideration that

$$\sum \epsilon = 0,$$

we obtain the following true reading:

$$\varphi = \frac{1}{4} (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)$$

As was first shown by Tserossky, the position of the natural star with respect to the artificial star has considerable influence upon the result of the measurement. This occurs because the images of the two stars when observed simultaneously fall on different parts of the retina of the eye the sensitivity of which to the sensations of light may not be everywhere the same. Therefore, it is best to do the equalization at the four positions of the natural star;

above and below, and to the right and to the left of the artificial star. This rule applies not only to Zellner's photometer, but to any photometer using an artificial star.

It appears from formula (13) that the error of brightness of the Zell photometer depends upon the angle φ . If we take the accuracy of the reading of the circle of intensities to be equal to $0^m.1$, then for various values of φ we shall obtain the following errors of brightness δ_m :

φ	5°	10°	15°	30°	45°	60°	75°	85°
δ_m	.043	.022	.014	.007	.004	.002	.001	.0003

It would seem that it would be more advantageous to observe at large values of φ . But, as practice shows, at large values of φ it is difficult to obtain an accurate reading because of the slow change in brightness of the artificial star. The best readings are made at an angle between 5° and 50° , where the mean error of the determination of brightness remains almost the same along series of observations made by Zellner's photometer at the Potsdam Observatory show the mean error of a determination to be $\pm 0^m.06$.

Pickering's Photometers. The use of an artificial star as an intermediate object of comparison is the cause of certain systematic errors arising from the unequal form of the natural stars and of the artificial. The form of the natural stars depends greatly on the condition of the atmosphere and changes from evening to evening. In general, the artificial star is always less bright than the natural one. Another principle is used in the photometers introduced by E. Pickering at the Harvard Observatory, namely, that of immediate comparison of two natural stars with each other. Of the numerous photometers devised by Pickering we shall describe only one, the meridian photometer. We already know that the very numerous observations made by means of this photometer in both hemispheres is the basis of the international system of visual stellar magnitudes.

The meridian photometer was given its name because it was developed for the determination of stellar magnitudes as the stars were in transit across the meridian. It consists of a long wooden box placed in the direction East to West. (see Figure 43). This box contains two tubes having objectives A and B, which differ slightly in focal length. In front of them are two mirrors C and D, inclined at an angle of 45° to the optical axes of the objective. The mirrors may be rotated around the optical axes by means of handles E and F. As the objectives are somewhat inclined with respect to one another, we may bring the two stars to the same point of the common field of view by a suitable turn of the mirrors.

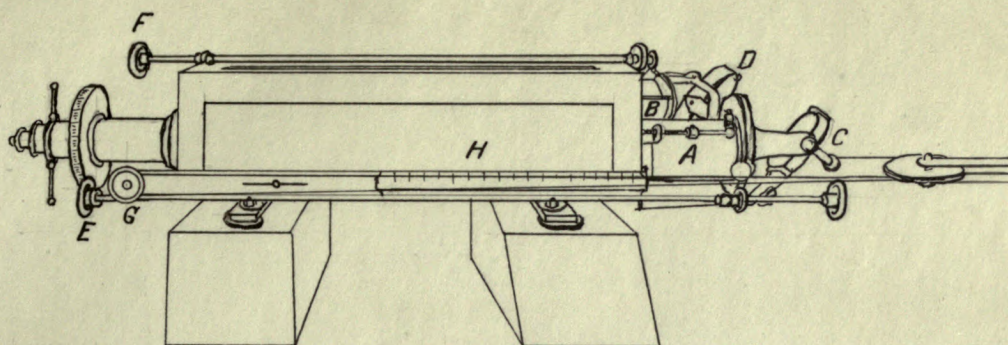


Fig. 43.

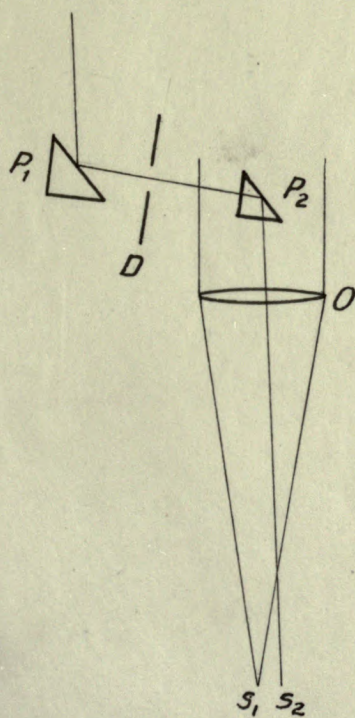


Fig. 44.

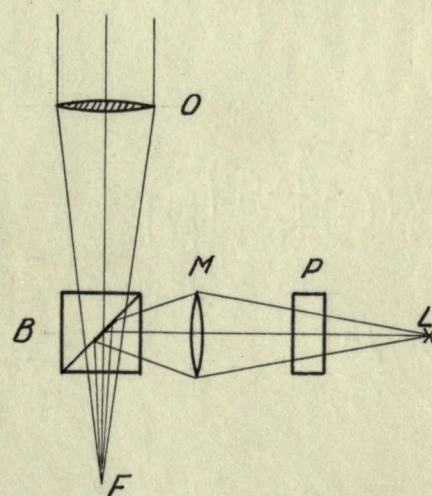


Fig. 45.

§ 14. The Photometric Properties of the Photographic Plate. The problem of photographic photometry consist in the determination of the brightness of the source of light by measuring the effect of the source of light upon a photographic plate. In photographing the stellar regions at the focus of the objective, the stars are represented as black disks of different diameters depending on the brightness. In photographs taken out of focus the stars have the form of disks of the same size depending upon the distance of the plate from the focus and the disk differ in their blackness. The images of these celestial bodies which have a noticeable angular diameter represent a different degree of blackening at the different points depending on the distribution of brightness on the surface of the celestial body.

It is immaterial for the purposes of photometry in what units the measured effect is expressed. In the case of extra-focal photographs the densities or the blackening of the negative are used. They may be determined in the following manner.

Let a ray of light of intensity I_0 fall upon the photographic plate. In passing through the layer of the plate the ray is somewhat absorbed so that its final intensity is I . The density of the image is determined from the following expression:

$$\frac{I_0}{I} = e^D,$$

where e is the base of the natural logarithms and D is the density. The blackening S is connected with the density by the following relation:

$$S = 0.4343 D$$

or

$$S = \log \frac{I_0}{I}$$

Let us have on the plate a series of images caused by the source of light the brightness of which has been changing in a definite manner. Let us plot the logarithms of brightness on the abscissa axis, and the corresponding densities on the ordinate axis. The obtained points, after they are connected by a continuous curve, represent the relation for the given plate between the density of the image and the intensity. This curve is called the characteristic curve and its general form is as that represented in Figure 46.

Three parts are to be differentiated on the characteristic curve. The first part which is turned with its convex side toward the x-axis is a region of under exposed images where the density grows very slowly as the intensity increases. Next, in the second part, the curve climbs steeply upward and it is nearly a straight line. Finally, in the third part, the curve becomes

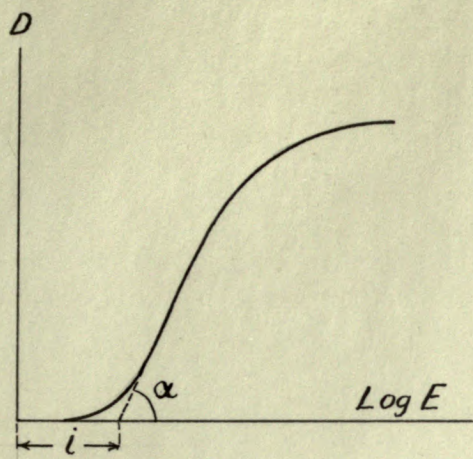


Fig. 46.

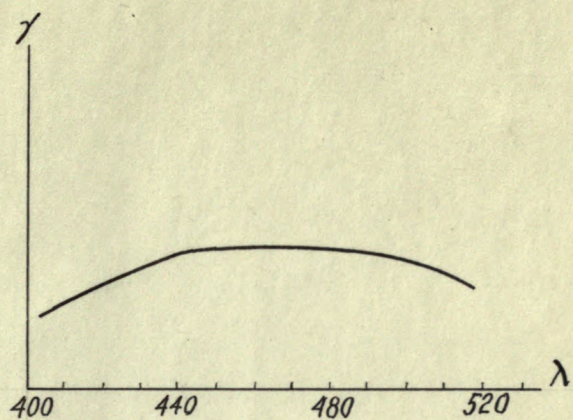


Fig. 47.

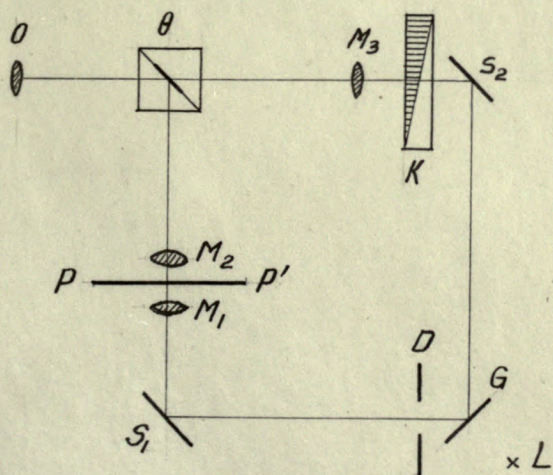


Fig. 48.

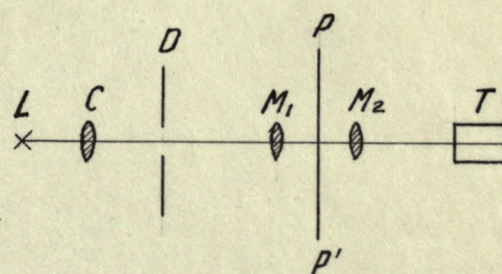


Fig. 50.

gradually concave with respect to the x-axis and turns into a straight line parallel to the x-axis; this is the region of over exposed images. For very great intensities the curve begins again to come down, and the density falls again to zero at the moment of the beginning of the process of solarisation. The characteristic curve has also the same general form if we increase the exposure with a constant source of light.

For photographic photometry the most important part of the characteristic curve is its middle part where the increase of density reaches its maximum. Let us extend its straight line part until it intersects the x-axis. The distance from the point of intersection to the origin of the coordinates is called the inertia of a plate. If we denote the inertia by i , the sensitivity of the plate is expressed by $C: i$, where C is a constant. The slope of the ascent of the curve is determined by $\tan \alpha$. It is usually designated by the letter γ (gamma); thus

$$\gamma = \operatorname{tg} \alpha = \frac{\Delta D}{\Delta \log E}.$$

The quantity γ is called the gradation of a plate. Evidently, the greater the gradation the more accurately may the acting intensity be determined, from the measured density for in this case the error in measuring the density will cause the least error in the sought intensity.

The form of the characteristic curve is different for the various kinds of plates. In general, the gradation decreases with the increase of sensitivity; of the greatest gradation are the diapositive plates which are of small sensitivity.

The method of development of the hidden images influences greatly the gradation of a plate. Thus, for instance, the increase of the time of the development leads to the increase of γ ; we may say that

$$\gamma_t = \gamma_{\infty} (1 - e^{-kt}),$$

where γ_{∞} is the gradation corresponding to a rather long development, e is the base of the natural logarithms, K is a constant for a given type of plates. The concentration of the developer and its temperature act similarly on the characteristic curve. Changing the conditions of development, it is possible to change the gradation within the certain limits. Usually the plates used for photometric purposes are developed under strictly standard conditions. The developing must be done in such a manner that the developer can act uniformly upon the entire area of the plate and that the products of the development, without remaining on any part of the plate are interchanged by a fresh developer. Otherwise the plate will be developed unevenly and the density of the brightest object may be abnormally small.

The purity of the emulsion and the uniformity of the sensitivity of the plate upon its entire area are very important factors in the accuracy of photometric measurements. Non-uniform sensitivity may be due to the fact that the glass upon which the emulsion is poured is not sufficiently flat. In this case the thickness of the layer that is sensitive to light and the quantity of silver bromide will be uneven on the various parts of the plate. According to Eberhard's investigations the error in the determination of brightness due to the cause described above is, on the average equal to $+ 0^m.05$, but cases are not unusual in which the error reaches $0^m.2$ or $0^m.3$. Therefore for greater accuracy the use of plates which were prepared on very flat mirror glass is recommended.

The photographic plate is of unequal sensitivity with respect to light of various wavelengths. The usual bromogelatin plates (non-orthochromatic) have a maximum sensitivity lying in the region of wavelengths between 420μ and 460μ . The extreme limits are 320μ and 500μ . The character of the curve of sensitivity is different for the various kinds of plates. For some plates it rises steeply to a maximum from the short wave side, for other plates the steeper part of the curve corresponds to the longer wavelengths.

The aspect and form of the characteristic curve depends greatly upon the wavelength of the incident light. This dependence is of great importance in spectrophotometry and therefore it will be treated in greater detail in the chapter on spectrophotometry later in this book. But even in ordinary photographic photometry where non-orthochromatic plates are used it should be studied because it is the cause of phenomenon which is analogous to Purkinje's phenomenon in visual photometry. Figure 47 shows the gradation depending on the wavelength in that region of the spectrum with which we must deal in the determinations of photographic stellar magnitudes.

As appears from the drawing, the gradation increases with the wavelength and reaches its maximum approximately at $\lambda = 440\mu$ after which it remains constant up to $\lambda = 490\mu$. Then it decreases as the wavelength increases. Therefore, if we are to photograph stars of different colors, the density of the images will grow at unequal rates and hence their relative brightnesses, determined by means of photography, will contain an error depending on the color.

The property of the plate to react unequally to the continuous and the interrupted exposure even if the common duration is the same, is of great importance for photographic photometry. It appears that the density of the image obtained as a result of a continuous exposure is always greater than for an interrupted exposure of the same duration.

Plates of the same sort differ greatly sometimes in their photometric properties. It even happens that plates take out from the same box, i.e. plates prepared from the same emulsion, show noticeable differences in general

sensitivity as well as in gradation, which fact may be explained by the difference in thickness of the layer sensitive to light. Likewise, the properties of the plate are influenced by the temperature and moisture of the air during observations, by the age of the plate and by the time elapsed between exposure and development. The influence of all these factors has not been fully investigated; therefore, in photometric investigations requiring the highest accuracy it is necessary to obtain, as much as possible, uniform conditions of work. The most important condition is successful photographic - photometric determinations is standard development in the sense of the composition of the developer, its concentration, the temperature, and the duration of the development.

The so called Eberhard effect is of great importance for the extra-focal method. This effect results from the fact that the density of the image depends not only on the quantity of light reacting upon the plate, but also upon the linear dimensions of the image (all other conditions remaining the same). The density at the center of the extra-focal disk is always less than at the limb. The magnitude of the effect depends on the density. It is evident that we shall obtain for two stars unequal differences of densities if on the two plates their two images have different diameters. Eberhard believed that the effect depended on the kind of developer, and he believed that he had found that the effect disappears when an iron developer is used. However later investigations showed that this effect appears also with an iron developer. According to Velenkov this effect reaches the $O^m.13$ stellar magnitude for plates of high sensitivity.

§15. The Measurement of the Photographic Effect. Hartmann's microphotometer, which is widely used in the practice of photographic photometry, is used for measuring the densities or blackenings of the extra-focal stellar images as well as for measuring the photographs of celestial bodies having noticeable linear dimensions, such as the moon, the planets, nebulae, etc. This microphotometer works according to the principle of photometric equalization of two adjacent illuminated fields and it is schematically represented in Figure 48.

A strong source of light L illuminates plate G of a ground or milky glass. The place to be measured on the negative PP' on one hand and the photometric wedge K on the other hand are illuminated by means of mirrors S_1 and S_2 by the scattered light of the above mentioned source. Between the plate G and mirror S_1 is placed the diaphragm D , in which the diameter of its aperture may change. Its real image is formed on the surface of the negative by the objective M_1 . The image of the portion of the negative to be measured, which is formed by the objective of microscope M_2 after the prism of Lummer-Broadhun is reflected from the little mirror S_1 and observed by means of eye-piece O . Simultaneously, in the same eye piece a portion of the wedge is seen whose image is given by objective M_2 . Thus, in the field of view of the eye-piece there will be seen the little mirror s of the prism illuminated by the light that has passed through the object under consideration, upon the background of wedge K . It is possible, by means of moving the wedge, to equalize the brightnesses of the two fields; at this moment the entire field of view will appear equally il-

luminated. It is then evident that the density of the image of the object to be measured is equal to the density of the corresponding place in the wedge. The brightness of the object is determined from the reading, made upon the scale fastened to it. The objectives M_2 and M_3 are equipped with moving rails which enable us to focus them upon the measured image and the wedge. We can now see quite well the granular structure of the image and the wedge (photographic wedges are used in Hartmann's microphotometer). This circumstance renders the work by means of this instrument more difficult, for it is necessary that the dimensions of the grain be approximately equal here as well as there, otherwise the evaluation of the photometric equality of the fields under comparison is inaccurately made. In order to avoid this the focusing of the two objectives is sometimes adjusted in the same degree so that the grain ceases to be visible. It is also important that the colors of the two fields be the same.

The accuracy reached in the measurements by means of the microphotometer is very high. When the wedge is carefully placed, and the mean error of measurement is only $0^m.01$ to $0^m.2$ of a stellar magnitude.

Diaphragm D_1 whose image is projected by objective M_1 upon the negative to be measured, serves for the adjustment of the sign of the pencil of light. Its dimensions must not be noticeably greater than the dimensions of the extra-focal image of stars. Otherwise, the light, having passed by the image and falling upon the objective, will reflect again upon the image of the star and will cause a systematic error in the density. As was shown by Schwarzschild, this error may reach under certain conditions to 50% of the measured intensity.

In the new constructions of Hartmann's microphotometer the measured plate is placed in a holder that may be moved by means of moving rails along two mutually perpendicular directions. The motion is read on scales. Thus, the instrument gives us, besides the blackening, also the approximate rectangular coordinates. Figure 49 shows the model of the microphotometer made by the Askania-Werke.

Hartmann's microphotometer is used not only for the measurement of extra-focal images of stars or luminous objects which have large angular dimensions, but also for the measurements of spectra. For this purpose the ordinary Lummer-Broadhun prism having a round little mirror is supplanted by another in which the little mirror has the form of a narrow slit that enables us to separate narrow bands in the spectrum to be measured.

The Microphotometers of Schildt and of Rosenberg. Besides Hartmann's microphotometer other instruments known as objective microphotometers are being introduced. They are so constructed that the human eye is supplanted by a sensitive thermoelement or photoelement.

The scheme of Schildt's microphotometer is very simple. The light of a

lamp L fills by means of a condenser C, the aperture of the diaphragm D (see Figure 50). The diminished image of this aperture is projected in the form of a brightly illuminated little area, upon the plate PP' to be measured. After having passed through it the light is gathered by objective M upon the surface of the sensitive thermoelement T. Under the influence of the radiant energy (falling upon the surface of the thermoelement), whose quantity depends on the transparency of the plate at the place of measurement, a thermo-electric current is generated which may be measured by a galvanometer.

Schildt's microphotometer has the advantage over Hartmann's in that it enables us to measure the extra focal as well as the focal photographs of stars. In the former case it is evident that the readings on the galvanometer will depend on the density of the extra focal image. In the latter case we set a focal image of a star within the luminous image of diaphragm D. Then the deviation of the little mirror of the galvanometer will be a function of the diameter of the image, as the quantity of light falling on the surface of the thermoelement is proportional to the difference of the squares of the diameters of the image on the plate of diaphragm D and the image of the star.

Due to convenient situation of the separate parts of the instruments and due to the fact that there is no need to make repeated observations on the same object because of the objective method of measuring the photographic effect, this instrument enables us to measure quickly a large number of stars. Of importance is also the fact that it enables us to measure even poor stellar images, such as those distorted by Coma, astigmatism, and generally blurred stellar images.

Rosenberg's microphotometer is based on the application of photoelements and is of a more complicated structure. Its scheme is represented in Figure 51. The light of a lamp L is concentrated by the condenser O_1 on diaphragm B whose form of aperture and dimensions may be changed. The image of this diaphragm, filled with the light, is formed on the surface of plate PP' by the objective O_2 . The light, having passed the portion of the plate to be measured is concentrated by objective O_3 , upon the little mirror of the Lummer-Broadhun prism W, is reflected from it, is collected by objective O_4 on the photometric wedge K, and after having passed through the wedge, falls on the layer, that is sensitive to light, of the photoelement Z_1 .

On the other hand, the light of lamp L falls upon the second photoelement Z_2 , having passed through the diaphragm "iris" J. The electric scheme is compensational in which the string electrometer E1 is used as a zero instrument. Because of the illumination of the photoelements a photocurrent begins in their circuit and the thread of the electrometer will deviate from its normal, zero position. For a certain mean intensity of the light falling on Z_1 - an intensity that corresponds to the mean position of the wedge K -, it is possible to bring the thread again to the zero position by means of an auxiliary compensational device K_2 . The change in the intensity of the illumination of the photo-

element Z_1 is due to the difference in density of the objects measured on the plate; it will cause the deviations of the thread of the electrometer. By moving the wedge they are brought to zero. Thus, in Rosenberg's microphotometer the photometric wedge is the measuring device. This instrument, as well as Schildt's microphotometer, is used for the measurements of focal as well as extra focal photographs.

The simplest method of measuring the photographic effect is the scale method. Let us take on the plate a series of photographs of a star setting them in a straight line at a certain distance one from the other. And let us increase the exposure from one to the other in a certain ratio. Then, after their development, we shall obtain a series of images of a continuously increasing photographic effect. If the photograph was taken in focus, we shall have a series of disks of gradually increasing diameters; in the case of an extra-focal photographs we shall have images of increasing density. Such a photograph is a scale of the photographic effect which we can use for the measurement of objects that are of interest to us by expressing their brightness in units of the scale.

§ 16. Relative Photographic Photometry. Let us assume that we had measured the photographic effect, caused on the plate by stars that we are interested in, by any of the methods described above. The essence of photographic photometry consists in finding methods of transfer from the measurement of the photographic effect (which may be expressed in quite arbitrary units) to photographic brightness. By photographic brightness we mean the integral brightness which corresponds to the region of sensitivity of the photographic plate. Keeping in mind what we said previously, we may derive the following formula for the photographic brightness of a star:

$$I = \int_{.32}^{.50} F(\lambda) \varphi(\lambda) d\lambda,$$

where $F(\lambda)$ is a function of the sensitivity of the plate and $\varphi(\lambda)$ is the law of the distribution of the brightness in the spectrum. It is evident that photographic brightnesses differ generally from visual ones. However, in stellar photometry it is conditionally accepted that the photographic and the visual brightnesses coincide for the white stars, or more accurately: the zero points of both systems coincide for stars of the spectral type A0 at the sixth magnitude on the visual scale. As to the scale of the photographic stellar magnitudes, we have here, as well as in visual photometry, the following relation:

$$m = -2.5 \log I + \text{const.}$$

There are two methods, different in principle, of photographic photometry: the absolute method, where a scale of stellar magnitudes is obtained independently of any brightnesses known in advance, and the relative method where the connection between the measured photographic effect and the brightness is based on the brightnesses of basic stars determined in advance.

In the relative photographic photometry the whole matter is reduced to the construction of the characteristic curve of the plate or to the derivation of the interpolational formula which is used for the computation of stellar brightnesses according to their diameters or densities. The international undertaking for the preparation of a photographic catalogue and map of the sky has greatly accelerated the development of the relative methods. On the plates, made for this purpose, were measured or evaluated besides the rectangular stellar coordinates, also the diameters of their images; the stellar magnitudes were then computed by the interpolational formulae. The measurement of the diameters is accompanied by peculiar systematic errors depending on the structure of focal images. Their limbs never appear sharp which fact depends, aside from atmospheric conditions, on the granular structure of the plate, and also on the degree of the correction of the aberration of the objective. It is therefore understandable why all attempts to derive a general formula giving the relation between the diameter and brightness suitable for all instruments have been unsuccessful. Various authors have obtained formulae of various forms; we shall give a few of them in what follows.

Kapteyn applied the following formula in his Cape "Durchmusterung" of the sky (C. P. D.) for the determination of the photographic magnitude according to the diameter d :

$$m = \frac{a}{b+d}$$

where a and b are constants for the given plate which are determined from known stars.

Scheiner's formula is written as follows:

$$m = a + bd,$$

and the diameter, according to Scheiner, may be expressed as a function of the exposure as follows:

$$d = d_0 \sqrt{t}.$$

According to Turner, the formula is

$$m = a + b\sqrt{d}.$$

It is hardly possible to give preference to any of these formulae. Scheiner's formula represents satisfactorily the measurements for small brightnesses and deviates greatly from the true interpolational curve for large diameters. On the contrary, Turner's formula represents poorly the measurements for small diameters. In general, none of these formulae is suitable for an interval of brightnesses that exceeds 3 stellar magnitudes.

The accuracy of the determination of a stellar photographic magnitude according to the diameter does not exceed $0^m.07$ and it depends, as is easily seen, upon the rate of growth of the diameter or upon the magnitude of the coefficient b in the formula of Scheiner and Turner. It is of interest to note here what at first sight may appear as a paradox, namely, that the accuracy of the determination of m according to d is less, the better the objective is i.e. the better the aberrations (chiefly spherical aberration and Coma) have been corrected in the objective. Therefore, the newer astrophotographic objectives that reduce aberrations to a minimum are less suitable for photometry according to diameters than the older objectives.

Direct measurements of the diameter are seldom made at the present time. In most cases objective microphotometers or the method of photometric scales are used for focal photometry.

Likewise, computations of stellar magnitudes by means of a formula are usually avoided, but instead the graphic relation between the measured effect and the magnitudes of basic stars already known is found. From the characteristic curve thus obtained the stellar magnitudes under consideration are derived.

In the case of the observation of variables a method similar to that of Pickering in visual photometry is used. For this purpose one or two pairs of comparison stars of known photographic magnitudes are selected near the variable. By means of simple interpolation between them we obtain directly m and it does not exceed $0^m.1$ of a stellar magnitude.

If the plate shows an insufficient number of stars that are necessary for the drawing of an interpolation curve, we must then take advantage of the method of double exposure. This method may be described as follows: we photograph directly on the same plate, one after the other, the region in which are situated the stars to be measured and another region of a sufficient number of stars that have been well measured. Both photographs (on the same plate) are taken at the same zenith distance and the same exposure. Moonless nights, in which the transparency of the atmosphere may be expected to be constant during the two exposures, are selected for such photographs. In most cases, however, a general fog appears on the plate as a result of the use of an objective of high light gathering power and of long exposures that are necessary in order to obtain images of faint stars. The fog is caused by the scattered light of the celestial background and it introduces complications. Indeed, the second photograph is already made on a somewhat fogged plate, and the densities of the stellar images are increased by the density of the fog which fact may cause an error in the scale of the stellar magnitudes. On the other hand it is known that a plate which has been subjected to a faint illumination increases its sensitivity, and an error in the zero point results.

One of the advantages of photographic photometry consists in the possibility of making determinations of stellar brightnesses of stars scattered over a large field. However, such determinations may not always be compared among themselves. It is because of the change in the character of the stellar images as they depart farther from the optical axis and because of the smaller or greater curvature of the focal surface that is a property of each given objective. The plate is usually set so that the sensitive layer is tangent to the focal surface at the center of the plate. Then the stars at the edges of the plate will not be in focus which fact will show itself in the magnitude of the photographic effect and will cause a systematic error in the determination of brightness. This systematic error depends on the position of the star with respect to the center of the plate. It is called field error and has a different magnitude depending on the properties of the instrument used. As an example we shall give here the corrections for the stellar magnitudes, depending on the distance from the center of the plate, for the astrograph of the Yerkes Observatory (see Table 4). The astrograph is equipped with a Zeiss doublet objective.

Table 4

Distance from the Center	Correction
0°.0	+ 0 ^m .00
0 .5	+ 0 .01
1 .0	+ 0 .03
1 .5	+ 0 .08
2 .0	+ 0 .14
2 .5	+ 0 .22
3 .0	+ 0 .32

Especially large is the field error for objectives and reflectors of high light gathering power and of short focus, where the curvature of the focal plane is quite large and the aberration grows rapidly as the distance

from the optical axis increases. Usually, for objectives free from the defects of centering and astigmatism, the field error may be represented in the following form

$$\delta n = c\rho^n,$$

where ρ is the distance from the center, c and n are constants whose size varies for stars of different brightness.

§ 17. The Relation between the Photographic Effect, the Brightness and the Exposure. We have previously said that the photographic effect depends, everything else being equal, upon the brightness and exposure. If we deal with densities then

$$D = \psi(d),$$

where d is the latent photographic effect for which we should write

$$d = \varphi(I, t).$$

If we know the analytic form of function φ it is evident that we could construct an absolute scale of photographic magnitudes by means of varying the duration of the exposure of the star field under investigation. It appears, however, that this method can never yield assured results because of the complexity of the photo-chemical processes that take place under the influence of light in the sensitive layer of the plate and because the properties of the plate are not constant. It is therefore but a relative method. Let us enumerate here a few of the results of the work devoted to this problem.

According to Bunsen and Roscoe the photographic effect depends upon the quantity of light energy absorbed by the layer of the plate, or

$$d = \varphi(I \times t)$$

Making this assumption, the following condition applies to the exposures in order to obtain images of equal density from two stars:

$$\frac{t_0}{t_1} = \frac{I_1}{I_2}$$

whence it follows that

$$I \times t = \text{const.}$$

This is the so called reciprocity law. This law has been subjected to numerous verifications by laboratory experiments and by astronomical observations, which have shown that this law does not hold. Indeed, as according to what has been said

$$\frac{I_1}{I_2} = 2.512^{m_2 - m_1},$$

one stellar magnitude should have been gained when the exposure is increased 2.512 times. In fact, it follows from direct photographic observations that the gain is much smaller and is of the order 0^m.7.

Schwarzschild has offered another formula for the change in the photographic effect. He proposed, on the basis of experimental considerations, that the following relation obtains between the density of the hidden image, the intensity, and the exposure:

$$d = It^p,$$

where p is an index less than unity and is constant for a given plate. Thus, unlike the reciprocity law, we shall obtain the same photographic effect only when the intensities and the exposures are subjected to the following condition:

$$It^p = \text{const.}$$

Applying this condition, as was done previously, to two stars, we shall obtain:

$$\left(\frac{t_2}{t_1}\right)^p = 2.512^{m_2 - m_1}.$$

Taking the logarithms of both sides we shall have

$$.4(m_2 - m_1) = p \log\left(\frac{t_2}{t_1}\right).$$

Hence, it follows that an increase of the exposure by 2.512 times gives us a gain of p stellar magnitudes. The index p in Schwarzschild's formula is easily determined by means of two stars of known stellar magnitude. Namely, we obtain:

$$p = \frac{.4(m_2 - m_1)}{\log\left(\frac{t_2}{t_1}\right)}$$

Here t_2 and t_1 are the exposures for which the densities or the diameters of the images of the two stars appear to be equal. If p is known for a given plate, the photographic magnitudes of the stars that are on the plate are easily determined.

Schwarzschild made extensive investigations for the determination of the value of p , and the result was that this value varies for various plates from 0.6 to 0.8. Later numerous investigations have disclosed the p is not constant even for the same plate, but changes within wide limits depending on the wavelength of the acting source of light and on its intensity. Therefore Schwarzschild's formula cannot serve as the basis of absolute photographic photometry.

Extensive investigations of the dependence between D , I , and t were made by Kron at the Observatory of Potsdam. Kron studied the densities obtained for intensities and exposures that changed within wide limits. Thus, the intensities changed in the ratio of 1:3000, and the exposure from 1 second to 8 days. The measured densities were used as data for the construction of curves of equal densities, i.e., curves that determine the relation between $\log I$ and $\log (It)$. The curves have a form as represented in Figure 52. The investigation of a series of such curves constructed for several values of the density has shown that the form of the curves does not depend on the absolute value of the density and it is nearly a hyperbola. It appeared next that for each of the investigated types of plates there exists a certain intensity for which the quantity of light $I.t$, that is needed to make a definite photographic effect on the plate, is a minimum. Kron has called such an intensity the optimal intensity.

As a result of his investigations Kron arrived at the following relation between the density, intensity, and exposure:

$$D = \varphi(d),$$

where

$$d = t \cdot I \cdot 10^{-a \sqrt{(\log \frac{I}{I_0})^2 - 1}}$$

where a is a constant for a given type of plate, I_0 is the optimal intensity. Thus, according to Kron, an equal photographic effect for stars of different brightness will obtain when the exposures are subjected to the following condition:

$$t \cdot I \cdot 10^{-a \sqrt{(\log \frac{I}{I_0})^2 - 1}} = \text{const.}$$

It is remarkable, that the law of reciprocity and Schwarzschild's law are special cases of Kron's formula. Thus if $I = I_0$, we shall obtain

$$I t = \text{const.}$$

On the other hand, if I is small as compared to I_0 , then neglecting unity under the square root in Kron's formula, we obtain

$$t \cdot I \cdot 10^{-a \log \frac{I}{I_0}} = t \cdot I^{1+a} = \text{const.}$$

or denoting

$$p = \frac{1}{1+a}$$

we shall obtain

$$I \cdot t^p = \text{const.}$$

Hence it follows that the law of reciprocity holds for intensities close to the optimal intensity. Schwarzschild's law holds for small intensities that do not exceed the optimal intensity. The region of application of the two laws in photographic photometry is limited by these conditions.

Kron made his investigations with four types of plates and obtained the following values of the constants a and I_0 , and the illuminations by Hefner's lamp at a distance of 1 meter was taken as a unity of intensity.

	a	I
Diapositive plates	Agfa 0.22	1.7
	Ideal 0.11	0.59
Highly sensitive plates	Schleussner 0.19	0.42
	Seed 27 0.21	0.13

The optimal intensity I depends on the duration of development, and I_0 is greater for all plates, the longer the duration of development.

Kron's formula is a large step forward on the road of the investigation of the law of photographic action, but, as it has no theoretical basis, it may be recognized only as an empirical formula which is suitable within wide limits of intensities and exposures.

§ 18. Methods of Absolute Photographic Photometry. In absolute photographic photometry the scale of stellar magnitudes should be obtained independently of any preassigned stellar brightnesses. In other words, the same special experiment.

This can be done in two ways. First, we may standardize the plate that is to be used for photometric work by laboratory means, i.e. we may obtain on it a series of images of an artificial source of light whose brightness changes in a known ratio. Second, it is possible to derive the characteristic curve from the photograph of the stellar region if we make two photographs on the same plate, diminishing the brightness of all stars from one photograph to the other in a certain ratio. The exposures should be of the same duration, and the devices used for the weakening of light should not change the spectral peculiarities of the source of light in the first case and that of the stars in the second case.

The laboratory standardization of the plate may be done either by obtaining on it images made with a well investigated wedge which must be strictly neutral, i.e. it must weaken equally the light of those wavelengths which also subsequent)

exposure in the so called tube photometer.

The tube photometer was introduced into astronomical practice by Parkhurst and it consists of several tubes which are blackened inside and whose entrance apertures are closed by round diaphragms of various diameters. The exit apertures are of the same diameter. The tubes are usually placed in a box in which may be placed the plate to be standardized; the plate is in contact with the exit apertures of the tubes. If we turn the front of the photometer toward scattered light, the illumination of the plate will be proportional to the area of the entrance apertures of the tubes. It is assumed that the diameters of the apertures are small compared with the length of the tubes. Upon development the plate will show a series of images of the exit apertures of various densities. Their corresponding intensities may be easily computed. Let the diameter of the largest diaphragm of the photometer be δ_1 . Let us take the corresponding intensity of the illumination of the plate, made by this aperture, as unity. Then the intensity for the image formed by some other tube of a diameter δ_n will be expressed as follows:

$$I_n = \left(\frac{\delta_n}{\delta_1} \right)^2,$$

and the difference of the stellar magnitudes will be:

$$\Delta m = -5(\log \delta_n - \log \delta_1).$$

Having measured the densities of the tube photometer, we built the characteristic curve by which we determine the stellar magnitudes of the stars photographed on the same plate at a certain distance from the focus. In order to avoid the Eberhard effect it is necessary that the diameters of the extra-focal images of stars be equal to the diameters of the images of the photometer. The method of tube photometry has become widespread, even if it is not free from shortcomings are: 1) the non-simultaneous photographing made under different temperatures; 2) difference in stellar color and in the source of the scattered light (Purkinje's effect). Often during observations of faint stars, it also becomes necessary to diminish by several times the exposure of the tube photometer as compared with the exposure of the stellar region. However, as experiment has shown, the summary influence of these sources of systematic errors in the scale is comparatively not very great and it is expressed as a very small fraction of the brightnesses under determination.

Let us now pass on to that group of methods of absolute photographic photometry in which the connection between the brightness and the photographic effect is obtained from the photograph of the stellar regions.

Let us have two photographs of any region of the sky on the same plate; the first is taken at a free aperture of the objective, the second, is taken when the stellar light is weakened by suitable means by k stellar magnitudes. The two photographs have the same exposure times. Let the two photographs be extra-focal and let it be required to find the interrelation between the density and the stellar magnitudes in the form

$$D = \varphi(m),$$

which may otherwise be written as follows:

$$m = \psi(D).$$

From the measurements we obtain in densities of the two images of each star on the plate: the normal image D and the weakened image D' . Thus

$$D = \varphi(m); \quad D' = \varphi(m+k).$$

Let us construct on the bases of these changes the so called density curve, plotting D along the abscissa and the corresponding D' along the ordinate. The equation of the curve will be

$$D' = \chi(D).$$

It is evident that the problem of constructing of a characteristic curve will be solved, if from the preassigned function χ we can determine the function φ .

On the bases of the foregoing equations we may write:

$$\varphi(m+k) = \chi[\varphi(m)].$$

Schwarzschild indicated that this expression is nothing else then one of the functional equations studied by Abel, and he gave its general solution as applicable to the conditions of photographic photometry. We shall give here the results of this solution for certain simple special cases which most often obtain in practice.

a) If the difference of the densities of the two photographs are constant, i.e.,

$$D' - D = c,$$

the equation above is satisfied by a linear relation, between the stellar magnitude and the density, of the following form:

$$m = \frac{k}{c} D.$$

b) If there exists a linear dependence, between the densities of the normal and weakened images, of the following form

$$D' - a = b(D - a),$$

where a and b are constants, the solution of the problem will be a logarithmic relation between m and D :

$$m = k \frac{\log(D - a)}{\log b}.$$

c) In the case when $D' = D^n$, we should have the following: $m = \frac{\log D}{\log n}$.

Thus we may construct a characteristic curve suitable for the entire interval of the measurable densities of stellar images. Naturally, all previous considerations hold also in the case of the measurement of the diameters.

We shall say nothing here about the practical methods of constructing the characteristic curve*). Let us enumerate the methods used in photographic photometry of the weakening of stellar brightness in a definite ratio.

1. The Diaphragm Method. Two photographs are taken: one through a free aperture of the objective of radius r_0 and the other by means of round diaphragm of radius r . Evidently, the weakening of light at the second photograph will be

$$K = 5 \log \frac{r_0}{r}.$$

This method has the following deficiency: when the objectives is covered by the diaphragm to a large extent, the character of the focal image changes at this circumstance may lead to systematic errors in the measurements of the image. Besides, for various diaphragms, the light passing through the thickness of the glass of the objective suffers uneven absorption, and hence, the purely geometric formula for the weakening of light requires a correction for the absorption in the lenses.

2. King's Method. Two photographs, taken at different distances of the plate from the focus, are compared. The stellar images will then be of different density. If we denote the illuminations which are obtained from a star, when the plate is moved by ΔF_1 and ΔF_2 , by i_1 and i_2 , then

$$\frac{i_1}{i_2} = \left(\frac{\Delta F_2}{\Delta F_1} \right)^2$$

and the difference of the stellar magnitudes will be:

$$\Delta m = 5 (\log \Delta F_2 - \log \Delta F_1).$$

*) An example of such a construction is given in the "Gottingen Actinometry" by Schwarzschild.

Because of zonal errors of the objective and its incomplete achromatization, the distribution of densities in an extra-focal image usually changes depending on the motion of the plate. Therefore, this method is used only with large ΔF when these deficiencies of the extra-focal images are evened up. This causes a great loss of light, and therefore the method can be used only for bright stars. It is necessary to keep in mind Eberhard's effect which is fully effective here.

3. The Metallic Net Method. The diaphragm method is applicable only for photographs taken in the focus of the objective, King's method, on the contrary, is applicable only for extra-focal photographs. The method of weakening of stellar light by means of a metallic net placed in front of the objective is applicable in both cases. The net gives a weakened stellar image surrounded by diffraction spectra. The ratio of the intensities of the central image and the image obtained without the net may be computed, if we know the thickness s of the wires and the width of the free interspace a :

$$\frac{I}{I_0} = \left(\frac{a}{a+s} \right)^4.$$

Hence the absorption made by the net is expressed in stellar magnitudes as follows:

$$\Delta m = 10 \log \frac{a}{a+s}.$$

The presence of the factor 10 shows that the measurements of a and s should be made with great accuracy if we wish to know the absorption of the net with an accuracy of one hundredth part of a magnitude. However, even very careful measurements may yield an erroneous result if the net is made with insufficient accuracy, i.e. if, for instance, the width of the interspaces or the thickness of the wires are not quite the same over the entire area of the net. Therefore, it is better to have the absorption of the net determined by the laboratory photometric method by means of observations of an artificial star. In so doing it is necessary to measure the absorption made by the entire net, and not by its separate parts, in order to avoid the influence of its local irregularities.

A variation of the net method is that proposed by Kapteyn and subsequently developed by Schwarzschild who called it the method of the half net. It consists in the following; between the objective and the plate, close to the latter, a thin metallic net is placed which covers only half the plate. Two photographs are taken: one without and the other with the net. It makes no difference whether they are taken on one or two plates. Let us denote the densities on the first plate by D_1 (m) and those on the second by D_2 (m). Let the absorption made by the net be K stellar magnitudes and let it be so situated that it covers the right hand half of the plate. Then we shall obtain from the measurements the following for the two stars one which is on the left hand side and the other is on the right hand side of the plate:

	Left hand half	Right hand half
Photograph without the net	$D_1 (m_1)$	$D_1 (m_2)$
Photograph with the net	$D_2 (m_1)$	$D_2 (m_2 + k)$

We form differences for all stars

$$D_1 (m_1) - D_2 (m_1) = \Delta D_2 (m_1)$$

and construct a curve giving us the relation of these differences from the density D_2 proper. The differences represent the influence of the change of all the conditions of photographing which have taken place between the first and the second photograph. By means of the curve we reduce all densities of the right hand half of the second photograph to the conditions of the first photograph. For the stars of the right hand half we shall then have densities corresponding to the stellar magnitudes m and $m + k$, as if these photographs were taken simultaneously under identical conditions.

One of the objections which may be raised against any method of obtaining an absolute scale of photographic magnitudes that requires two photographs, - namely, their non simultaneousness, - does not apply to the half net method. The quite possible fluctuations of the transparency of the air between the two photographs, especially in long exposures, render the laboratory determinations of the factor k not equal to its true value. Likewise, the change in temperature of the sensitive layer and the influence of moisture cause changes in the photometric properties of the plate. In the following methods, where the two photographs, the normal and the weakened, are obtained simultaneously, this objection is removed.

4. The Method of the Diffraction Grating. This method was proposed by Hartzprung and it consists in the following. In front of the objective is placed a grating consisting of parallel bands or wires which are stretched on the frame at equal distances one from the other. Then each star on the plate is represented in the form of a central disk on both sides of which will be situated diffraction spectra of various orders. As is known from the theory of the diffraction grating, the distance R of the ray of wavelength λ from the central image is:

$$R = \frac{F}{a+s} \lambda,$$

where F is the focal length of the objective and $(a + s)$ is the constant of the grating. The latter may be so selected that the spectra of the first order have (without bending with the central image) a negligible length not exceeding the limiting diameter of stars.

The distribution of light between the central image and the spectra of various orders may be easily computed. If I is the intensity of the image without the grating, I_0 is the intensity of the central image, and I_n is the intensity of the spectrum of the n -th order, then

$$\frac{I_0}{I} = \left(\frac{a}{a+s}\right)^2; \quad \frac{I_n}{I} = \frac{1}{(n\pi)^2} \cdot \sin^2\left(\frac{an\pi}{a+s}\right).$$

Assuming that $I = 1$, we shall obtain the following in stellar magnitudes:

$$m_0 = 5 \log \frac{a+s}{a}, \text{ and}$$

$$m_n - m_0 = 5 \left[\log \left(\frac{an\pi}{a+s} \right) - \log \sin \left(\frac{an\pi}{a+s} \right) \right].$$

If the grating is so constructed that $a = s$, then we shall have for the spectra of the first order (which are quite sufficient in photographic photometry) the following:

$$m_0 = 1^m.50; \quad m_1 - m_0 = 5 \log \frac{\pi}{2} = 0^m.98.$$

Thus, such a grating which is called a normal grating, weakens the stellar light by $1^m.50$. Of considerable importance is the difference in brightness between the central and the first diffraction images. This difference is the basis for the construction of the characteristic curve. The general absorption of such a grating as composed with the normal photograph is equal to $2^m.5$, which fact necessitates longer exposures. However, this shortcoming is overbalanced by the fact that the grating method requires only one exposure in which are obtained simultaneously two images of each star whose brightness ratios may be accurately computed in advance. Besides, this method is equally applicable to focal as well as extra-focal photographs.

5. The Method of the Prismatic Satellite also requires no double exposure. In front of the objective of the astrograph is placed a small prism of very small refracting angle. Then there will appear on the plate,

at a certain distance from each star, an image of a satellite formed by the rays deviated by the prism. Due to the small refracting angle of the prism the stretching of the satellite is not noticeable to the eye. If the radius of the aperture of the objective is denoted by r_0 , the radius of the prism and its mounting are denoted respectively by r and c , the difference in brightness is stellar magnitudes between the normal image and the satellite will be:

$$k = 2.5 [\log (r^2 - c^2) - \log r^2].$$

This formula derived from purely geometric considerations does not give us an entirely accurate value of k because no consideration is given to the absorption and reflection of the rays of light in the prism and the objective. To take these factors into consideration rigorously is very difficult.

4. Electrophotometry

§ 19. Introduction. Electrophotometry is one of the youngest branches of practical astrophysics. The beginnings of electrophotometry are due to J. Stebbin's work of only about twenty years ago. Developing comparatively slowly because of the novelty of the methods and the difficulties connected with the measurements of weak electric changes and circuits, nevertheless, electrophotometry has already obtained a noted position among the methods of quantitative study of radiant energy of celestial bodies. There is no doubt that a great future belongs to it because it represents an objective method in the sense that it frees our eye from any judgment on the brightness of the objects under investigation or on the magnitude of the effects caused by them on a plate. It remains for the eye only to record the readings of the electrical measuring device. Aside from this tremendous advantage over visual and photographic photometry, electrophotometry has immediately shown such a high degree of accuracy in the obtained results that there has opened before it a very wide field of investigations in the region of the study of very small fluctuations of brightness of celestial objects. The magnitude of these fluctuations is so small that their detection lies almost on the borderline of possibility, which is determined by the presence of irregular and incalculable changes in the transparency of the terrestrial atmosphere.

Electrophotometry in its application to astronomy is based on the utilization of two physical phenomena, namely: 1) the change in electrical conductivity of certain substances under the influence of the radiant energy that falls upon them, and 2) the phenomenon of photoelectricity in the narrow sense of the word. Only in recent years there appeared the possibility of utilizing a third kind of photo-effect for photometric purposes namely: the excitation of the electromotive force in the thermo-couples made of certain metals or their oxides, under the influence of light. This effect

apparently reaches its greatest force in the so called copperoxide elements. However, the technique of their preparation as well as the method of utilization of the effect itself have not been sufficiently studied. Therefore we shall speak here only of the devices based on the photoeffects of the first two kinds mentioned above.

§ 20. The Properties of Selenium. Stebbin's Photometer. The photoelectric effect that expresses itself in the change of electrical conductivity or resistance of certain substances under the influence of light has been known for a long time and it has been observed in selenium, tellurium, sulphur, silver bromide, silver chlorid, and others. Among them selenium occupies an exceptional position because of the magnitude of its photoeffect.

There are two forms of selenium: amorphous and crystalline forms. The amorphous selenium is a complete insulator and is not sensitive to light. By means of a slow increase in temperature amorphous selenium passes into crystalline selenium and obtains electrical conductivity which changes upon the illumination of its surface.

The methods by which selenium gets its maximum sensitivity to light is a trade secret of the firms which prepare the selenium photoelements. Outwardly the photoelement is a plate made of an insulating material. Around this plate is wound closely a double row of thin metal wire. One of the surfaces of the plate is covered by a thin layer of molten selenium which was subject to a long processing by heat at a temperature of 190° to 210°C (the temperature of melting for selenium is 217°C). The selenium element, prepared in this manner, has a great resistance, the magnitude of which depends on the method of preparation. In the better selenium elements the resistance is of the order of 10^6 ohms. When the electrodes of the thermo couple are connected to a galvanic cell, a current appears in the circuit the force of which, all other conditions being equal, depends on the intensity of the light falling on the element. From the photometric point of view, it is important first to know the law of the change in the resistance of the element depending on the force of illumination upon the surface of the element. Unfortunately, various elements behave in this respect, differently. For the better elements it is close to a linear relation within wide limits of intensities. In any case, the given element must be carefully calibrated.

As to the color sensitivity, again do the various elements differ one from the other. Nevertheless, it is possible to state that the maximum sensitivity of selenium lies in the region of red rays. On the average, the maximum is at $\lambda = 700\mu$, and, as a rule, the color sensitivity falls rapidly on both sides of this value.

The resistance of selenium changes not only under the influence of illumination, but also to a great extent under the influence of the tem-

perature. Thus, Stebbins found that the resistance of the element then at his disposal changed in complete darkness from 1.10^6 to 3.10^6 ohms as the temperature changed from $+20^\circ\text{C}$ to 0° . As the temperature falls the sensitivity of light increases. This property greatly increases the difficulties in the application of selenium for astrophotometric purposes.

In similar manner the resistance of selenium changes when it is included in the electric circuit. In the first moments the resistance rapidly diminishes reaching a constant magnitude only after about half-hour. This phenomenon is apparently due to the thermal action of the current passing through the windings of the photoelement. It also makes necessary the closing of the circuit long before photometric measurements are to be made.

The other important properties of selenium are its inertia and fatigue. If we illumine a selenium photoelement, its resistance does not immediately reach a magnitude corresponding to the force of illumination, but reaches it only after a certain interval of time. Likewise, when the illumination ceases, the original degree of resistance is reached but gradually. Figure 53 is graphic representation of the change in the electrical conductivity of the selenium element upon its illumination and cessation of illumination. The time required for the establishment of a constant resistance in both cases is of the order of several minutes. Because of these properties selenium cannot be used when the illumination changes very rapidly, and in any case the utmost care must be given to the time element when we use selenium.

Finally, it is necessary to mention that the humidity of the air has a great influence upon resistance of selenium.

Selenium elements have been in use for some time for various technical purposes. But attempts to apply them to astrophotometry as light receptors were accompanied by numerous difficulties arising from the fact that, with the faint stellar illuminations with which we deal in astronomy, the photo-effect is of the same order, or even of a smaller order, as the various influences, upon the resistance of selenium. Therefore, it was necessary to construct the photometric device and to develop the methods of observation in such a manner that all these factors would have no harmful influence upon the results of the measurements.

Stebbins solved this problem in the following manner. His photometer consists of a small camera of double walls, and it is fastened to the eyepiece end of the refractor. The space between the double walls is filled with melting ice. Inside the camera, where a constant temperature is thus obtained, is placed the selenium element. In order to keep dry the inner part of the camera, the latter is connected with vessels containing substances which absorb moisture. The front part of the camera has a small aperture which is closed by a thin transparent plate in front of which is placed a shutter. Outwardly the camera is covered by a substance that is a poor conductor of heat. (Figure 54).

The scheme of the inclusion of the selenium element into the electric circuit consists of connecting the selenium cell in one of the branches of a Wheatstone Bridge. Other branches are supplied with variable resistances used to compensate the whole apparatus in the darkness and enabling it to reach the most advantageous conditions of work for the sensitive galvanometer which serves for the measurement of the force of the current in the bridge during the illumination of the photoelement. A battery of 6 volts was used by Stebbins as the source of electromotive force.

The currents to be measured are very weak. Stebbins worked with a 12 inch refractor and the currents even from bright stars were of the order of 10^{-10} to 10^{-12} amperes. Therefore the entire electric scheme must be entirely insulated and protected from electrostatic influences. The sensitivity of the d'Arsonval's galvanometer used by Stebbins was such that 1mm on the scale at a distance of 1m corresponded to a current of 24×10^{-10} amperes.

All the electrical devices are placed in a separate compartment; hence, two observers are needed for the work with the selenium photometer. One observer sets the refractor upon the star to be measured while the other is occupied with the readings of the galvanometer. The observation proper consists of projecting on to the surface of the selenium the out of focus image of a star which was 7mm in diameter in Stebbins' case, and the exposure lasted for 10 sec. Then a "rest" of one minute was given to the selenium.

Stebbins applied the described set up for the study of the variable Algol (β Persei), and he was the first to establish indubitably the existence of a secondary minimum which reaches only $0^m.08$. The accuracy of the measurement of the difference of brightness between Algol and the comparison star was about 2%.

It is necessary to admit that observations with a selenium photometer are complex and require much time. Thus, 20 minutes were required by Stebbins for a complete observation. For this reason use of the selenium photometer has not become widespread; also the fact that selenium has a comparatively small sensitivity limits this instrument. Thus Stebbins, using a 12 inch refractor, was able to go only as far as $3^m.5$.

Recently Bernheimer showed that selenium photoelements of a different kind, namely, condenser photoelements, are of much greater sensitivity to light than those used by Stebbins. These elements are of very small size: the diameter of the area covered with selenium is equal to one square millimeter. Bernheimer proposed to use two selenium elements of which one serves as a receptor of light and the other, placed near the first, is included as a compensation resistance in one of the branches of a Wheatstone Bridge. Thus the temperature effect is destroyed. Bernheimer used the 40 centimeter reflector of the Vienna Observatory and was able to obtain a measurable photoeffect of stars of 8-9th magnitude.

§ 21. Alkaline Photoelements. Almost simultaneously with Stebbins' work, experiments in the application of another photoeffect to astrophotometry were begun. The other photoeffect was discovered by Hallwachs who noticed that a metallic plate loses its negative charge when it is illuminated by ultra-violet rays. It was subsequently found that the alkaline metals such as potassium, sodium, rubidium, and caesium show this effect very strongly. The physical essence of the phenomenon consists in the fact that the energy of the light that is absorbed by the surface layer of the metal frees a certain number of electrons which leave the metal with great velocities. The basic law of the photoelectric effect states that the quantity of free electrons is proportional to the intensity of the light and to the area of the illuminated surface of the metal. The alkaline metals show the effect not only under the influence of ultra-violet light, but their sensitivity extends over a quite wide range of wavelengths including also a portion of the visible spectrum.

If we place near the surface of an alkaline element a metallic plate and connect them by a conductor, a current will run through it upon illumination. This current is called a photo-current. Such a scheme is the principle of the construction of photoelements. As the alkaline metals rapidly oxidize in the presence of the oxygen in the air, the two electrodes are placed in a vacuum glass bulb (or sometimes in a quartz bulb).

There exist numerous types of photoelements of which the spherical photoelements are almost exclusively used in astronomical practice. One of the halves of the spherical bulb is covered inside by a layer of an alkaline metal which is in contact with an electrode that comes outside. Not far from the layer is a wire ring or a thin grid which also has an outside connection for the current. Both leads are to be used for the connection of the photoelement to the measuring circuit.

Such vacuum photoelements are comparatively of little sensitivity. However, the photocurrent may be greatly increased by including a battery in series with the photoelement; the cathode of the battery is connected with the surface of the alkaline metal and the anode with the ring or the grid. The electrons, which fly off the surface of the metal at various angles under the influence of the electrical field, will rush toward the positively charged ring; and the greater the difference of potential, the greater will be the numbers of those moving electrons. However, the increase in the difference of potential within the evacuated bulb is advantageous only up to a certain limit (from 50 to 100 volts) at which all electrons reach the ring and the photocurrent reaches a degree of saturation.

In astrophotometry, where the illuminations by stellar light are weak, an increase in the sensitivity of photoelements is of great importance.

The technique of their preparation follows two methods of increase in sensitivity. The first consists in the excitation of a smouldering discharge inside the bulb filled with hydrogen. After the hydrogen is removed the metal, which is now in a colloidal condition, is many times more sensitive to light than before this process, called sensitizing, was applied. The second method consists in filling the bulb after sensitizing by one of the inert gases (helium, argon) under a slow pressure. The photo-current in such a photoelement filled with gas will be greater under the same illumination than in a vacuum photoelement, because with a sufficiently large difference in potential, the photo-electrons receive so much energy that they are able to ionize the gas by colliding with the molecules. Thus, the number of free electrons, flowing to the anode, increases and the photo-current becomes greater.

This phenomenon is of great importance. Evidently the degree of ionization of the gas will continuously increase, as the difference in potential at the binding screws of the photoelement increases. It seems that by raising the difference in potential we could increase the photo-current indefinitely. However, at a certain difference in potential, whose magnitude depends on the nature of the gas and its pressure, a discharge takes place in the photoelement and its qualities deteriorate or the photoelement may even be entirely destroyed. Near such a critical potential, which may reach from 200 to 300 volts, the photoelement obtains its maximum sensitivity. A too close approach to the critical potential is not advantageous, because the element under such an approach shows indications of becoming "fatigued", these indications are exactly the same as those appearing in the case of selenium.

While working with photoelements filled with gas, the question arises as to whether the basic law of a photoeffect — namely, the proportionality of the photo-current to the illumination, — still holds. It appears that in a carefully prepared photoelement this law holds; nevertheless, each photoelement should be carefully calibrated before it is used for photometric purposes.

The photoelements have the property of a selective photo-effect. The various alkaline metals are not equally sensitive to light of various wavelengths. Usually, for a given metal there exists a wavelength for which the emission of photo-electrons reaches a maximum value. In the following table are given the wavelengths which correspond to the maximum emission for a series of metals:

Sodium	340 μ
Potassium	440 μ
Rubidium	480 μ
Cesium	560 μ

Because of this phenomenon the photoelements may be applied to the quantitative measurement of radiant energy in various portions of the spectrum.

Precautions for the prevention of disturbing influences upon the photo-effect should be taken in the construction of the photoelement and while working with it. Especially harmful are the parasitic currents which spread between electrodes upon the inner and the outer surfaces of the glass bulb. To prevent them, the exits of the cathode and the anode are usually surrounded by metallic rings which are grounded during the measurement. These currents are one of the causes of the appearance of the so called "currents of darkness" in the photoelement.

As an example of external causes disturbing the photo-effect we may mention the influence of the electro-magnetic field of the Earth upon the paths of the photo-electrons inside the element. This influence is noticeable in astronomical observations when the photometric device is fastened to the tube of the refractor or reflector which changes its position with respect to the magnetic axis of the Earth.

The photo-effect may be accurately measured by various methods which may be divided into two basic groups. To the first group belong such methods where the force of the photo-current, which originates in the circuit under the influence of illumination on the photo-element, is measured directly by the galvanometer. However, these methods are not quite suitable for astronomical observations, as the photo-currents are small because of the weakness of the stellar illumination given to the photoelement and they are contained within the limits from 10^{-7} to 10^{-15} ampere. Of greater advantage is the other group of methods based on the measurement of weak charges by means of electrometers. For this purpose string electrometers are usually used which are of small capacity. The most sensitive in this group is the method of measuring the time necessary for the charge of the electrometer to build up to a certain definite potential. The scheme of the connections of photoelement Z, battery B, and the electrometer E is represented in Figure 55. Under the influence of the illumination of the photoelement the thread of the electrometer receives a charge which reaches the magnitude V during the time t. If we denote by C the capacity of the electrometer, the force of the photo-current is determined from the following formula:

$$i = \frac{C}{t} \frac{V}{t},$$

where C is the value of the unit of capacity - the farad - expressed in the CGS system ($C = 9 \times 10^9$). Having determined the time during which the thread of the electrometer deviates from its normal position by the same number of divisions on the scale for various illuminations of the photoelement, we obtain the force of the current by this formula. The ratio of the intensities of two sources of light may be found from the following relation:

$$I_1 : I_2 = t_2 : t_1.$$

This method has great sensitivity which is the greater, the smaller c

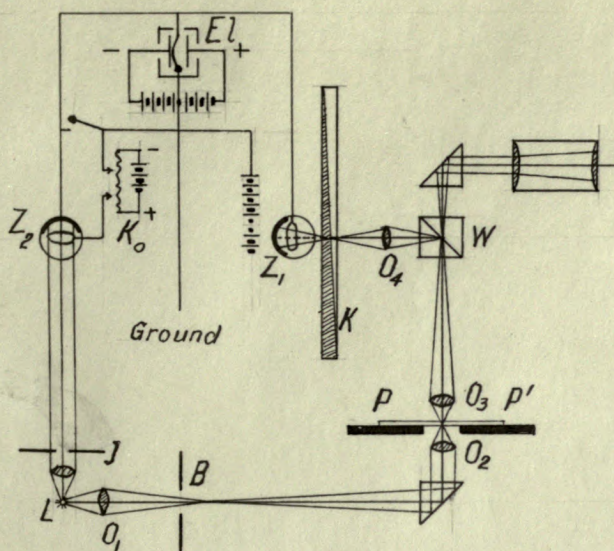


Fig. 51.

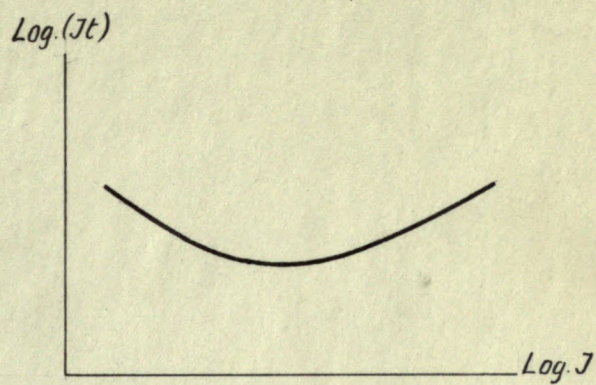


Fig. 52.

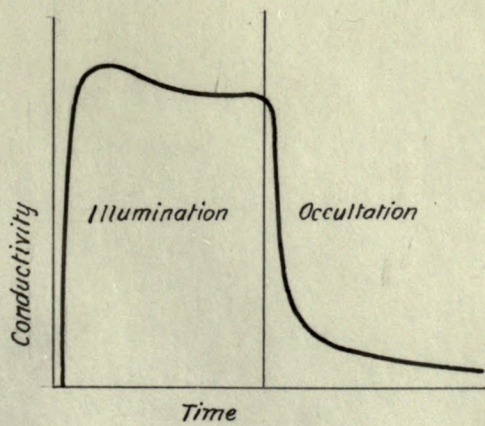


Fig. 53.

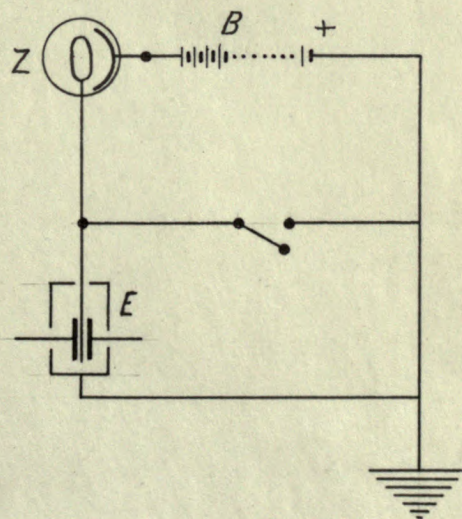


Fig. 55.

is, all other conditions being equal. In contemporary electrometers the capacity is of the order of but a few centimeters. On the other hand, this method is very sensitive to the change of capacity of the entire electrical apparatus. It is very difficult to keep it unchanged under the conditions of astrophotometric determinations when the position of the various parts of the photoelectric scheme changes together with the position of the tube.

§ 22. Guthnick's Stellar Photometer. There are at present several models of electrophotometers which differ from each other but in a few unimportant details. We shall therefore describe here in detail only one of them, namely, the photometer constructed for the Berlin Observatory at Babelsberg according to Guthnick's instructions.

The outward appearance of the photometer is shown on Figure 56; the position of its separate parts are shown on Figure 57.

The photometer is attached to the eye piece end of the tube by the ring AA. The focal image of a star is obtained in the plane BB where the iris diaphragm is situated; it is used to prevent the entrance of light into the photoelement from the background of the sky and from other stars close to the object to be measured. The photoelement is situated at a distance of 20cm. from the focus, and thus a large extra focal disk is obtained on its sensitive layer. In the path of the rays a rectangular prism C may be placed which directs the rays into tube D. At the eye piece there is obtained an image of the aperture of the iris diaphragm in whose center the star, which is to be measured, is placed. The angular diameter of the aperture of the diaphragm is made equal to $1' - 2'$.

The surface of the hypotenuse of the prism is silvered and therefore the light of the artificial source set at various distances from E may be directed upon the photoelement by means of window E which is closed by ground glass. Next follow the light filter FF and shutter GG. The described parts of the photometer may rotate around the optical axis of the instrument by means of rings aa and bb. The photoelement M is placed in an iron box K which has a cylindrical form. Its walls are grounded and serve as a protection from electromagnetic influences for the photoelement. L contains a small vessel of sodium which absorbs the humidity of the air that surrounds the photoelement. From the cathode a wire comes outside through a quartz plate N. This exit is also dried by sodium which is placed at P. The anode is connected through an aperture in the quartz O to the electrometer. The photoelement proper is equipped with protective metallic rings which are connected by a conductor with the Earth and which prevent the development of creeping currents.

Wulff's String Electrometer is suspended in gimbals S and therefore it maintains automatically a vertical position at any position of the tube. The method of the connection of the anodal clamp O with the thread of the

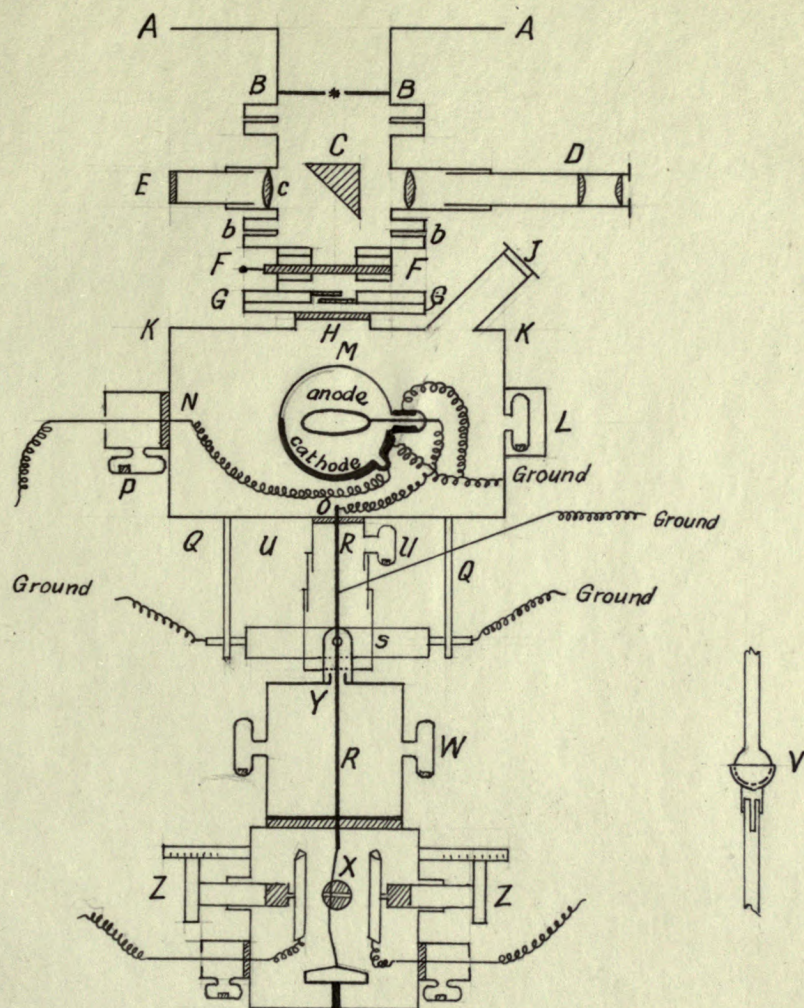


Fig. 57.

The Wulff String Electrophotometer.

electrometer X requires our utmost attention. In Guthnick's photometer this connection is made by means of a thick conductor RR consisting of two parts which are connected with each other by means of ball joint which guarantees a dependable contact and does not prevent the electrometer from assuming a vertical position. This connection is shown separately at the right hand side of drawing 57, and is denoted by V. The conductor RR is protected from outside electrical influence by a metallic jointed flexible tube. From it the conductor goes to a key which makes possible a connection of the thread of the electrometer with the Earth. In order to protect it from temperature influences the electrometer W is surrounded by a substance which is a poor conductor of heat.

The method of observation by the electrophotometer consists in the following: after the star is placed at the center of the aperture of the iris diaphragm and after the thread of the electrometer is grounded, a reading on the scale of the electrometer is made and the zero point is determined. Immediately afterward the shutter GG is opened, the thread is disconnected from the Earth, and the time is registered on a chronograph. Next we observe the motion of the thread of the electrometer in the field of view of the microscope and we register the times of its passing through equidistant divisions on the scale. Finally, the thread is grounded again and a second determination of the zero point is made.

Let N_1 and N_2 be the zero points of an electrometer determined at the beginning and the end of the observation, T_1 and T_2 be the times of their determination, t_1 and t_2 be the time of passing of the thread across the two equidistant lines on the scale, s — the length of the path on the scale. Then the apparent brightness of the star is expressed as follows:

$$l = K \frac{s - \frac{N_2 - N_1}{T_2 - T_1}(t_2 - t_1)}{t_2 - t_1},$$

where K is the coefficient of proportionality which disappears upon the determination of the difference of the stellar magnitudes of two stars.

This formula holds under two conditions: 1) the velocity of the thread is proportional to the brightness; 2) the change of the zero point of the electrometer is proportional to the time. According to the special investigations made by Guthnick, both of these conditions are fulfilled in his photometer sufficiently well. Thus, for instance, having measured the brightnesses of the artificial source of light placed at various distances from window E and having compared the results with the computations based on the law of inverse proportionality to the square of the distance, Guthnick found that the deviations in the interval of brightness of $2^m.5$ are, on the average equal to $0^m.002$.

As to the zero point, its changes were very small.

Among the errors peculiar to electrophotometric measurements the errors due to the following influences are of importance: the influence of the background of the sky, the changes in the capacity of the instrument, the action of electromagnetic field of the earth, leakages due to the imperfection of the insulation, and the changes in the transparency of the atmosphere. By using the best insulation material around the conductors, by their suitable placement, and by a dependable grounding of the electrical scheme Gathnick was able to reduce the harmful electrical effects to insignificantly small amounts.

The error due to the background of the sky may reach a noticeable size especially on moon lit nights. As an example let us give the influence of the background of the sky upon the measurement of the differences in brightnesses for γ and A Orionis:

Date	Coordinates of the Moon the Moon		Influence of the background of the sky
1917 February 3	6 ^h 3 ^m	+ 24°	0 ^m .109
1917 February 4	7 6	+ 21	0 .154
1917 February 5	8 4	+ 18	0 .076
1917 February 8	10 7	+ 4	0 .021
1917 March 1	5 5	+ 25	0 .028
1917 March 4	8 0	+ 20	0 .017

The most difficult job in the reduction of photoelectric measurements is the computation of the weakening of light, due to the absorption by the terrestrial atmosphere and the overcoming of difficulties connected with the irregular (and often periodic) fluctuations of the transparency of the atmosphere. The fluctuations take place sometimes even on nights which are perfect from the point of view of the usual visual and photographic observations. They are very troublesome when a very high accuracy of photo-electric measurements is attempted. The only way to reduce their influence is a suitable distribution of observations. Gathnick recommends making a comparison of two stars a and b in the following sequence: ababa, and the separate measurements should follow one another as quickly as possible. Hence

following the requirement that the compared stars should be comparatively close to each other.

The difficulties of taking into consideration the differential absorption of light by the atmosphere are determined by the fact that each photo-element has its own sensitivity to color. As the coefficient of transparency of the terrestrial atmosphere is different for stars of various colors (or spectral types), it is evidently necessary to obtain from the photo-electric measurements is attempted themselves the special coefficients for the conversion from Muller's table of absorption of light to the absorption which is suitable for the given photo-element and the given stellar spectral type. In Table 5 are given the values of such coefficients for one of the sodium photo-elements used by Guthnick.

Table 5

Spectrum	Coefficient
BO	2.2
AO	2.0
FO	1.8
GO	1.65
KO	1.5
MO	1.4

As to the accuracy of the results obtained with his photometer, Guthnick investigated it in various ways and came to the conclusion that the mean error of one determination of the difference in brightness of two stars is, in general, less than $+0^m.01$. Under exceptionally favorable conditions it is only equal to $+0^m.005$. Such favorable conditions are: a small difference in brightness of the stars compared, their closeness to each other, and an ideal state of the atmosphere.

5. Visual and Photographic Catalogues

§ 23. The Photometric System. There are always certain systematic deviations between the measurements of stellar magnitude, visual as well as photographic, made by one of the methods described in the previous

sections. Even when the measurements and their reductions are made with all possible care, the deviations nevertheless much exceed the accidental errors of measurements. They are either of a personal character which are very noticeable in visual photometry or depend on differences in the instruments and of the conditions of observation. Therefore each large series of photometric determinations or a catalogue of stellar magnitudes represents a certain system determined by three constants which distinguish it from other catalogues.

If we compare the magnitudes of the same stars in two catalogues K_1 and K_2 , the difference between them may be usually represented by the following simple expression:

$$K_2 - K_1 = \alpha + \beta(m - m_0) + \gamma c.$$

where α is the difference in the zero points of the catalogues under comparison

β is the relative difference of the photometric scales, γ is the coefficient depending on the stellar colors which are usually expressed by certain numbers C . The combination of these three constants determines the system of a given photometric catalogue. The system is called an absolute system when the scale of the stellar magnitudes is determined independently of any pre-assigned stellar brightnesses. The zero point of a catalogue is always conditional and relative. It appears in certain cases that the formula given above is not sufficient for the representation of the differences between catalogues. For instance, terms may appear in them which depend on the second order of the arguments $m - m_0$ and C .

Likewise, when the determination of magnitudes extends to large sections of the sky or over the entire sky, they sometimes depend on the positions of the stars. The existence of such phenomena renders the study of catalogues very difficult.

Having made the above general remarks, we shall now proceed to the enumeration and characterization of the main catalogues, visual as well as photographic, of stellar magnitudes.

§ 24. Visual Catalogues. We shall not say much about the old photometric catalogues obtained on the basis of simple evaluations by the unaided eye. Although their inner accuracy is small (see § 8) and not always independent from each other, nevertheless they are still of interest, aside from historical significance, in that they are useful for the solution of the problem of secular changes in stellar brightness. Zimmer made an attempt to use them for this purpose, but his results are not convincing. Evidently the period of time that is covered by photometric observation is still too small. Pickering made an interesting investigation of the color equation of the old catalogues and compared them with the new ones. A color equation, according to Pickering is the difference in stellar magnitudes of spectral types of K0 and A0 which are obtained after taking into consideration the relative difference of the photometric scales of catalogues. Taking the Harvard system as the basic system, Pickering found that the old visual catalogues is

in nowise differ from the new ones in the sense of the color equation. Hence we may conclude that the color sensitivity of the human eye has not noticeably changed from a period of about 2000 years.

The most complete catalogue of visual stellar magnitudes is the Bonner Durchmusterung of the northern sky (B.D.). It contains approximate positions and evaluations by the eye 324189 stars from the North Pole to -2° of declination. The work was done, by Argelander from 1855 to 1861 by means of a 4 inch comet finder at the Bonn Observatory. Later Argelander's work was contained by Schonfeld up to -23° in declination and also at Cordoba (argentine) up to -62° in declination. The latter catalogues is denoted by C. D. (Cordoba Durchmusterung).

The intrinsic accuracy of these catalogues as compared with the newest photometric catalogues is not great. Thus, the probable error in the B. D. is, in the mean, equal to $+0^m.2$. However, these catalogues are the chief sources of visual magnitudes from the brightest stars to those of 9th and 10th magnitudes. Because they contain a great number of faint stars covering almost three quarters of the entire celestial sphere, these catalogues are of such great importance for stellar statistic that they have been often carefully studied to systematic errors of the stellar brightnesses contained therein. Such studies have been made by Pickering, Seeliger, and Pannkoek and as a result large tables have been constructed for reducing the stellar magnitudes of these catalogues to a homogeneous photometric system. As an example we offer a little table which gives the value of magnitudes $9^m.3$ according to the B. D. in the Harvard system for various hours of right ascension according to Pickering's investigations (declinations from $+41^\circ$ to 90°):

0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
$9^m.7$	$9^m.7$	$9^m.$	$9^m.7$	$9^m.7$	$9^m.7$	$9^m.7$	$9^m.7$	$9^m.7$	$9^m.8$	$9^m.9$	$10^m.0$
12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
$10^m.0$	$10^m.0$	$9^m.9$	$9^m.9$	$9^m.8$	$9^m.8$	$9^m.8$	$9^m.8$	$9^m.8$	$9^m.8$	$9^m.7$	$9^m.7$

After the invention of photometers a series of more or less extensive lists of visual stellar magnitudes was published in the form of catalogues. Such, for instance, are the *Uranometria Oxoniensis* which contains visual magnitudes of stars visible with the unaided eye from $+10^\circ$ up to the North Pole according to measurements with the wedge photometer, the catalogues of Peirce, Wolff, Ceraski which were derived from measurements with a Zellner photometer. But the most extensive series of photometric observations, which represent the basis of our contemporary knowledge of stellar visual brightnesses, were made at the Harvard Observatory by Pickering and his collaborators and at the Potsdam Observatory by Müller and Kempf. The results of the colossal work performed at Harvard were published in numerous articles in the *Annals of Harvard College Observatory*. Most observations were made by meridian photometers (2.4 and 12 inch) and they encompass the entire sky from the North to the South Poles. The results, published in the various volumes of the *Annals*, were the data of which Pickering composed a final catalogue of all stars up to $6^m.5$. It was published in the 50th volume of the *Annals*. The system of the catalogue, which bears the name of the Harvard Revised Photometry or H. R. P., is now recognized as the standard system of visual stellar magnitudes. The catalogue is distinguished by its high accuracy and uniformity. Among the Harvard Catalogues are also catalogues which contain very faint stars. A complete enumeration of all the Harvard Catalogues with a brief description of their contents appears in the 75th volume of the *Annals*.

The Potsdam Photometric Durchmusterung (Potsdamer Photometrische Durchmusterung or P. D.) is another example of an absolute photometric catalogue of high accuracy. It contains all the stars down to $7^m.5$ according to the B. D. from the equation of the North Pole and it is derived from observations made by Zellner photometers. Besides the visual magnitudes of 14199 stars, the catalogue gives also the evaluations of their colors according to a scale of fourteen steps denoted by various combinations of the letters: W (white) G (yellow), and R (red). The intrinsic accuracy of the P. D. is characterized by a probable error of $\pm 0^m.04$.

There is a noticeable systematic discrepancy between the Potsdam and the Harvard systems. This discrepancy is represented, according to Müller and Kempf, by the following formula:

$$\begin{aligned} \text{Potsdam} - \text{Harvard} &= + 0^m.229 + 0^m.024 (m - 2^m.25) - 0^m.027 \\ &\quad F - 0^m.008 (m - 2.25) F, \end{aligned}$$

where F is a number expressing the color of a star; for purely white stars (W) $F = 0$, for yellow-red where red predominates (RG -) $F = 14$. According to the formula the following differences in the sense of Potsdam - Harvard appear for stars of $8^m.00$ to $8^m.49$, for instance, depending on the color:

Color	N	GW	WG	G
Difference	+ 0 ^m .37	+ 0 ^m .30	+ 0 ^m .07	- 0 ^m .08

The P. D. system was subsequently extended to faint stars down to 9^m.5 according to the B. D. from + 80° to the North Pole. This catalogue is denoted by P. D. P.

At the Kharkov Observatory V. G. Pesenkov determined with the Zellner photometer 115 stars down to 9^m.0 from + 79° .5 to + 90.

§ 25. Photographic Catalogues. Passing to the photographic magnitudes, it is necessary to note that at present we still have no such catalogues in this field as the H. R. P. or P. D. Only the Cape Photographic Durchmusterung (C. P. D), containing evaluations of photographic stellar brightnesses in the region from - 18° to the South Pole, may be compared with the B. D. or C. D. Nevertheless, the sources of photographic stellar magnitudes are quite numerous. The first catalogue of absolute photographic magnitudes was compiled by Schwarzschild and is known under the name of "Gottinger Aktinometrie". It contains 3522 stars down to 7^m.5 of visual magnitude in the zone from 0° to + 20°. This work was done with the aid of a camera equipped with a special plate holder invented by Schwarzschild. This plate holder moves automatically during the exposure in two directions. These motions are so designed that the stellar images are obtained in the form of uniformly blackened little squares which are easily measured with a Hartmann microphotometer. Thereby was avoided the usual non-uniformity of the stellar extra-focal images. The catalogue is of very high accuracy. The probable error of the catalogue magnitudes does not exceed, in the mean, + 0^m.03.

The Yerkes Actinometry also contains stars down to 7^m.5 (visual). Its first part (Y. A. I) contains the stars of the region from + 75° to the North Pole; its second part (Y. A. II) contains stars in the zone from + 60 to + 75°. The first part of the Yerkes Catalogue, developed by Parkhurst, contains also the photo-visual magnitudes of the same stars determined upon ortho-chromatic plates with a yellow light filter in the second part (Y. A. II) are given only the photographic magnitudes. The work at the Yerkes Observatory is carried on with the same instrument and according to the same plane, and the absolute scale is obtained by means of the tube photometer; the zero point is determined from the white stars of the P. D.

The zone from + 57° .5 to + 77° .5 was worked by Leman - Balanovskaia at the Pulkovo Observatory, and it contains all stars of the P. D. As at the Yerkes Observatory, a tube photometer was used for obtaining the scale of stellar magnitudes. The accuracy of the three catalogues mentioned is almost the same. The probable error is, on the average about + 0^m.04.

Work is going on at present at the Harvard Observatory on the determination of photographic magnitudes of all stars down to 8^m.25 from the North Pole to the South Pole.

All the catalogues mentioned above refer to the brighter stars. The construction of an absolute system of photographic magnitudes of fainter stars is very difficult. It was first undertaken at the Harvard Observatory. In the 71st. volume of the Annals of the Harvard College Observatory were published. Leavitt's results on the determination of the photographic magnitudes of 96 stars situated close to the North Pole and forming the so called North Polar Sequence on N. P. S; the sequence contains stars from the 3rd to the 21st magnitudes. This system of photographic magnitudes was long considered as basic and it was extended at Harvard to separate regions uniformly distributed on the sky known as the Harvard Standard Regions (H.S.R.). The results of the determination of the photographic stellar magnitudes in these regions are published in H. A. volume 71. Besides, having in mind the reduction of the brightness evaluations to a uniform system, -- an effort made by various observatories in the measurement of plates for the celestial catalogue and map (Carte du Ciel), -- the Harvard Observatory contributed standard stellar magnitudes down to $15^m.0$ in 108 regions (H. A. volume 85).

A similar system -- the South Polar Sequence -- was developed at the Cape Observatory for the southern hemisphere.

Besides the Harvard Observatory, the determination of standard stellar magnitudes near the North Pole was also made at Greenwich and Ptolema. A final derivation of the system was made by F. Seares at Mount Wilson. Seares subjected all the hitherto made determinations of stellar brightness of the N. P. S. to careful analysis and criticism and obtained a new their photographic magnitudes using the instruments of Mount Wilson. The result of this great work is a catalogue of 96 stars published in the first volume of the publications of the International Astronomical Union (I.A.U.). Seares' system is now the internationally recognized system. The photometry of stars in the 139 regions -- the so called Kapteyn's Selected Areas -- situated north of -15° , is the extension of this system to other regions. Here Seares gives the photographic magnitudes of all stars up to $18^m.0$. Work is being finished at present on the determinations of the photo-visual stellar magnitudes in the same regions.

A comparison with the North Polar Sequence is an excellent method of the determination of relative photographic magnitudes. Such relative catalogues are: 1) a catalogue of stars brighter than the $9^m.0$ between $+65^\circ.0$ and the pole compiled by Dyson from the observations made at the Greenwich Observatory; 2) a catalogue of 8986 stars between $+40^\circ$ and $+45^\circ$ derived by S. Beliaevsky from the observations of the S. Beliaevsky from the observations of the Simeis Observatory.

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